**Spiders and a Fly**
Three spiders and a wingless fly crawl along the edges of a wired 3-dimensional cube. They can always see each other. The speed of each spider is at least one third of the fly's speed. Show that the spiders can catch the fly.

**Solution:** The spiders can catch the fly. Here is one of their possible strategies. One spider controls the edge $AB$ and the second spider edge $CG$, so that the fly cannot pass through any of the vertices $A$, $B$, $C$, or $G$. To control edge $AB$ a spider simply walks to this edge. If the fly is on the edge the spider moves towards the fly and presumably the fly moves away. The distance between $AB$ in the rest of the cube is $3 \times |AB|$ while 3 times the spider's speed is at least the fly's speed. After some initial period, the spider can ensure that its distance to $A$ (resp $B$) is always at most $1/3$ of the fly's distance, in the cube minus $AB$, to $A$ (resp. $B$).
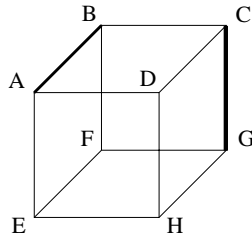


Then the remaining part of the cube does not contains any closed circuit and the third spider can easily catch the fly. This is the case of a spider chasing a fly on a tree. It simply walks towards the fly, which poor thing, eventually gets trapped at a leaf and eaten.