Spiders and a Fly
Three spiders and a wingless fly crawl along the edges of a wired 3-dimensional cube. They can always see each other. The speed of each spider is at least one third of the fly’s speed. Show that the spiders can catch the fly.

Solution: The spiders can catch the fly. Here is one of their possible strategies. One spider controls the edge $AB$ and the second spider edge $CG$, so that the fly cannot pass through any of the vertices $A$, $B$, $C$, or $G$. To control edge $AB$ a spider simply walks to this edge. If the fly is on the edge the spider moves towards the fly and presumably the fly moves away. The distance between $AB$ in the rest of the cube is $3 \times |AB|$ while $3$ times the spider’s speed is at least the fly's speed. After some initial period, the spider can ensure that its distance to $A$ (resp $B$) is always at most $1/3$ of the fly’s distance, in the cube minus $AB$, to $A$ (resp. $B$).

Then the remaining part of the cube does not contain any closed circuit and the third spider can easily catch the fly. This is the case of a spider chasing a fly on a tree. It simply walks towards the fly, which poor thing, eventually gets trapped at a leaf and eaten.

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