Puzzle 6: Unform Candy Distribution

n children are sitting around a circular table. Each child starts out with an integer number of candies. The following step is repeated:
Every child who has an odd number of candies is given another piece of candy by the teacher. Each child now has an even number. Now every child passes half of his/her candy to the child on his/her left.
Prove that eventually all the children will have the same amount of candy.

Solution
Consider the number of candies per person. Let these be $\{a_1, a_2, a_3, ..., a_N\}$ at a particular Stage, say Stage T.

(A): We can assume that these numbers are even after the numerical adjustment made by the Teacher.

Let the maximum number of candies held by one child in this stage be a_M, where the child is at the M'th position.
After the "pass-to-left-child" move the new numbers will be :

$$(a_N + a_1)/2, (a_1 + a_2)/2, ..., (a_{M-1} + a_M)/2, (a_M + a_{M+1})/2, ..., (a_{N-1} + a_N)/2.$$

Let the maximum in this stage be $(a_{K-1} + a_K)/2$.
We have

$$\begin{aligned} a_{K-1} &\leq a_M \\ a_K &\leq a_M. \end{aligned}$$

Thus $(a_{K-1} + a_K)/2 \leq a_M$.
Since a_{K-1} and a_K are even – **(A)**, they differ by at least a value of 2, or can be equal. If equal, $(a_{K-1} + a_K)/2$ is also even, and at most equal to a_M; if they differ by a value of at least 2, $(a_{K-1} + a_K)/2 < a_M$.
Thus, we have the following conclusion:

The value of the maximum either decreases, or remains the same. It will never be more than the original maximum (plus one, if this is odd).

Thus eventually, the teacher will stop handing out candy.

Now consider the change in the sum of squares of the number of candies held by each child.

This is

$$
(a_1^2 + \cdots + a_N^2) - \frac{1}{4}((a_N + a_1)^2 + \cdots + (a_{N-1} + a_N)^2) =
$$
$$
\frac{1}{2}((a_1 - a_2)^2 + \cdots + (a_{N-1} - a_N)^2).
$$

So, if the candy distribution is unequal, the sum of squares will decrease by at least one. So after a finite number of steps, the candy distribution is equal and the candy allocations do not change from that time on.

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