Solution: Erdős will win this game.
We provide two solutions. Many people produced something very similar to the following:
Notation: If integer x is the N digit number $\sum_{i=0}^{N} d_i 10^i$ with $d_N \neq 0$ then we write $x = x' + x''$ where $n = \lfloor N/2 \rfloor$, i.e. $x' = \sum_{i=0}^{n} d_i 10^i$ and $x'' = x - x'$. We can assume that $d_0 \neq 0$. Once $d_0 = d_1 = \cdots = d_t = 0$ then Erdős can maintain this.
Given x and $N > 2$, Erdős will present Oleg with $y = x'' + 10^{n+1} - x'$. Oleg's choices are $2x'' + 10^{n+1}$ or $|2x' - 10^{n+1}|$. In either case we can replace N by $\leq \lceil (N+1)/2 \rceil$. After at most 11 rounds, we will have $N \leq 2$.
When $N = 2$ we essentially are dealing with $a = 10x + y$, $x, y \neq 0$ and after one more round we find $a = 10(x+1)$ or $|10 - 2y|$. In both cases $N = 1$ and we can easily finish in 8 more rounds, e.g. by following the strategy proposed below:

The following solution is a little different and is due to David Wagner.
Say that a number x is n-nice if it can be written as $x = (p_1 + ... + p_n)/n$ where $|p_j|$ is a power of 10 for $j = 1, 2, \ldots, n$

Lemma 1 *With r rounds left to go, Erdős can guarantee a win if the current value a_{20-r} is 2^r-nice.*

Proof By induction on r. $r = 0$ is trivial.
Suppose it's true for r, and suppose a is 2^{r+1}-nice. Then a can be written as the average $a = (q + q')/2$, where q and q' are both 2^r-nice and where $q \geq q'$. If Erdős is smart (and he is), he will choose $b = q - a$ which is non-negative. Now Oleg is stuck. If Oleg chooses to add then he gets to $a + b = q$, which by induction lets Erdős win in r rounds. If Oleg chooses to subtract then he gets to $|a - b| = |q'|$, which is also 2^r-nice and thus by induction lets Erdős win in r rounds. Either way Oleg loses after $r + 1$ rounds. □

Lemma 2 *Every legal choice for a_1 is 2^{20}-nice.*

Proof Let $A = 2^{20} a_1$. Then A is non-negative and has at most 1008 digits. It can be expressed as a sum of at most 9×1008 powers of 10: suppose the i-th digit in the decimal expansion of A is d_i, so that $A = \sum_i d_i 10^i$; then writing $d_i 10^i = 10^i + ... + 10^i$ (with d_i terms in the sum) shows that A can be expressed as a sum of at most 9×1007 powers of 10, say n of them.
If n is odd, we can make it even by using the relation $10^{i+1} = 10^i + ... + 10^i$ (with 10 terms) or the relation 1 = 10 - 1 - 1 - ... - 1. This replaces 1 term in the sum with 10 terms, so the number of terms in the sum increases from n to $n + 9$.
After all this, we can arrange for A to be expressed as the sum of n powers of 10 (or their negations), where $n \leq 9 \times 1008$ and n is even.
Finally, pad out the sum to ensure we have exactly 2^{20} terms in the sum by adding $p(2^{20} - n)/2$ times and subtracting p the same number of times, where p is any power of ten. □

Acknowledgement: We thank Tim Clifford, Wenjie Fu, Karthik Lakshmanan, Victor Miller, C. Raptopoulos, Michael Schuresko, Sai Venkateswaran, David Wagner for sending in solutions.