Here comes Santa

Santa has been working hard all year and is now waiting at the bottom left corner \((0,0)\) of Gridville, sacks loaded with presents, ready to deliver them to the children waiting at the top edge \(\{(n + 1, x) : 0 \leq x \leq n\}\). Santa laid off too many elves last year and now things are running late. There are only \(H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\) days to Xmas. The Grinch see his chance to mess things up. Santa’s reindeer love Magic Reindeer Food (made from sugar, oatmeal and glitter) and the Grinch sprinkles \(F_{i,j}\) days worth of food on grid square \((i,j)\) for \(1 \leq i, j \leq n\). Magic Reindeer Food is in short supply and the Grinch can only afford to spread one days supply on each row i.e. \(\sum_{j=1}^{n} F_{i,j} = 1\) for \(1 \leq i \leq n\).

If Santa’s journey takes him to square \((i,j)\) then he will have to spend \(F_{i,j}\) of a day there while the reindeer’s eat the food that the Grinch has left.

Santa starts at \((1,1)\) and waits \(F_{1,1}\) days and then moves to \((2,1)\) or \((2,2)\). In general, if Santa is at \((i,j)\), then after waiting \(F_{i,j}\) of a day, he moves to one of \((i + 1, j - 1)\), \((i + 1, j)\), \((i + 1, j + 1)\). (Of course if \(j = 1\) or \(n\) only 2 of these moves are allowed).

Santa can make all his deliveries after reaching a square \((n,j)\) (and waiting \(F_{n,j}\)) because he can mount a non-deterministic delivery service from this time on. Can he make it in time, or will the Grinch ruin Xmas?