CG
10/18/12

Triangulating a PSLG.

Last time: $O(n \log n)$ algorithm

The $n \lg n$ came from sorting by y values.

Question: Do we need to sort?

Answer: No Chazelle $O(n)$ algorithm

Prob: Complicated.

Def: $\log^{(0)} n = n \quad \log^{(i+1)} = \log(\log^{(i)} n)$

for $n > 0 \quad \log^* n = \max_{l} \text{ s.t. } \log^{(l)} n \geq 1$

<u>Thm</u> G is a connected PSLG then can triangulate G in $O(n \log^* n)$ expected time.

Thm G connected PSLG then find the Trap Decom in $O(n \lg^* n)$ expected time.

Lemma Trap Decom to Monotone Decom to Triangulation is $O(n)$ time

Note: Seidel does horizonal extension.

Revisit our old random incremental Trap Alg.

Def $S = \{s_1 \cdots s_n\}$ n line segs

$\mathcal{T}(S)$ Trap decomp of S.

$\mathcal{Q}(S)$ point location data structure from last lecture.

For $s \in S$

$\deg(s, \mathcal{T}(S))$ = # of T in $\mathcal{T}(S)$ abuting interior of s.

Lemma $\text{Exp}(\deg(s, \mathcal{T}(S))) \le 4$

pf # ext = $2i$ abutting at most 2 seg

$$\sum_{s \in S} \deg(s, \mathcal{T}(S)) \le 4n$$

$$\text{Exp}(\deg(s, \mathcal{T}(S))) \le 4n/n = 4$$

Lemma 1) $s_1, \dots, s_n$ in random order of segs S

2) $S_i = \{s_1, \dots, s_i\}$

3) $\text{Trap}(S_i)$

4) $\mathcal{Q}(S_i)$ incremental Point Location DS (PLDS)

a) Expected size of $\mathcal{Q}(S)$ $O(n)$

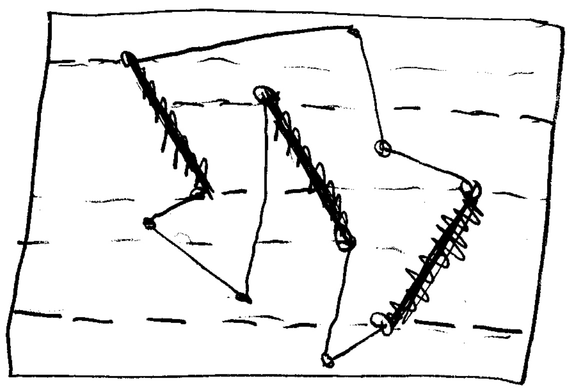
b) Expected comparison to locate q in $\mathcal{T}(S)$ using $\mathcal{Q}(S)$ is $\le 5H_n$

c) cost to build is $O(n \lg n)$

<u>Note</u> The expensive part is the point location using $Q(S_i)$.

Since G is connected we could also use $\Upsilon(S_i)$ to locate an end point.

eg



Assuming we have the Trap for red edges

Find missing endpoints by "tracing" paths.

<u>Def</u> $N(h) = \lceil n / \log^{(h)} n \rceil$

$N(0) = n/n = 1$ $N(2) = n/\log\log n$

$N(1) = n/\log n$ $\vdots$

Alg Trap(S)

1) Random order $s_1, \ldots, s_n$ of G

2) Init: $\mathcal{T}(s_1)$ & $Q(s_1)$

3) for $h = 1$ to $\log^* n$ do

   3a) for $N(h-1) < i \leq N(h)$

   Build $\mathcal{T}_i$ & Q_i from $\mathcal{T}_{i-1}$ & Q_{i-1}

   by inserting s_i.

   3b) ∀ endpoints $p \in$ {points not yet inserted}

   Trace p in $\mathcal{T}_{N(h)}$

4) For $N(\log^* n) < i \leq n$ do

   Finish using standard incremental.

pictures

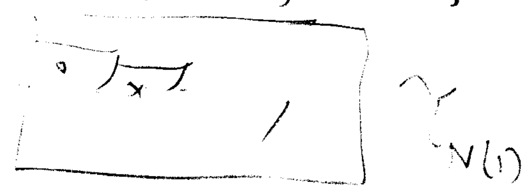
$N(0) = 1$

Tim

$N(1) = \frac{n}{\lg n}$

$N(2) = \frac{n}{\ell\ell n}$

Assign missing points to traps using Tracing



Lemma 4 $S_1 \sim S_n$ random order

Given τ_j & Q_j, remaining point in Q_j

Expect Query Time Q_K in $O(\log(K/j))$

Expected cost to Trace

Lemma $S \equiv n$-seg ; $R \subset S$ of r random seg

Let Z = # intersections of ext of $\mathcal{T}(R)$ and seg in $S \setminus R$

then $\mathrm{Expect}(Z) \le 4(n-r)$

Def $T \subset S$, $s \in T$ $\quad \deg(s, \mathcal{T}(T))$ = ext abutting s in $\mathcal{T}(T)$.

$$\sum_{s \in T} \deg(s, \mathcal{T}(T)) \le 4|T|$$

Note $R \subseteq S$, $s \notin R$ then ext of $\mathcal{T}(R)$ intersecting s is

$$\deg(s, \mathcal{T}(R \cup \{s\}))$$

$$E(Z) = \frac{1}{\binom{n}{r}} \sum_{\substack{R \subseteq S \\ |R| = r}} \sum_{s \in S \setminus R} \deg(s, \mathcal{T}(R \cup \{s\})) \quad (*)$$

$$(*) = \frac{1}{\binom{n}{r}} \sum_{\substack{R' \subset S \\ |R'| = r+1}} \sum_{s \in R'} \deg(s, T(R'))$$

$$\leq \frac{1}{\binom{n}{r}} \sum_{\substack{R' \subset S \\ |R'| = r+1}} 4|R'| = 4(r+1) \frac{\binom{n}{r+1}}{\binom{n}{r}}$$

$$= 4(r+1) \frac{r!\,(n-r)!}{(r+1)!\,(n-r-1)!} = 4(n-r)$$

Expect time for $T_{rap}(S)$

1) $O(n)$

2) $O(1)$

3) (3b) $O(n)$ time by last lemma

(3a) Query time:

$O(\log(i/N(h-1)))$ for $N(h-1) < i \leq N(h)$

$$\log\left(\frac{i}{N(h-1)}\right) = \log\left(\frac{i \log^{(h-1)} n}{n}\right) \leq \log(\log^{(h-1)} n) = \log^{(h)} n$$

$\frac{n}{\log^{h} n}$ queries costing $O(\log^{(h)} n) = O(n)$

3) total cost $O(n \log^* n)$

4) $N(\log^* n) \geq n/e$

$\therefore$ Expected point location cost $= O(1)$
Total $O(n)$