CG 1
11/27/12

Convexifying a Cycle (Part 2)

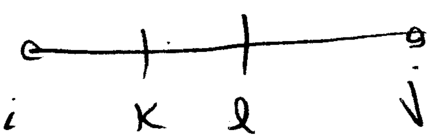
Def A stress is outer-zero if

$w_{ij} \neq 0 \Rightarrow e_{ij} \in$ Convex Cycle or inter to Con Cycle

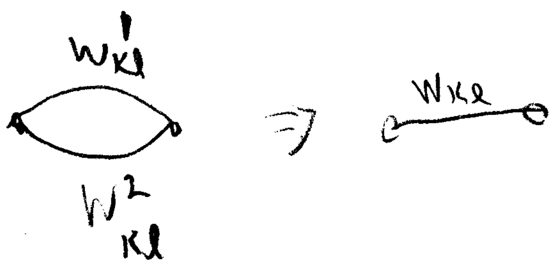
Def A stress $w_{ij} \neq 0$, $e_{ij} \notin CC$ is called outer-nonzero.

Lemma $\exists\, w$ of $G_A(P)$, w is nonzero proper equilibrium then $\exists\, w'$ of $G'_A(P')$ outer-nonzero proper equil stress.

Pf strut



$$w'_{kl} = w_{ij}\frac{|P_i-P_j|}{|P_k-P_l|}$$



$$w_{kl} = w^1_{kl} + w^2_{kl}$$

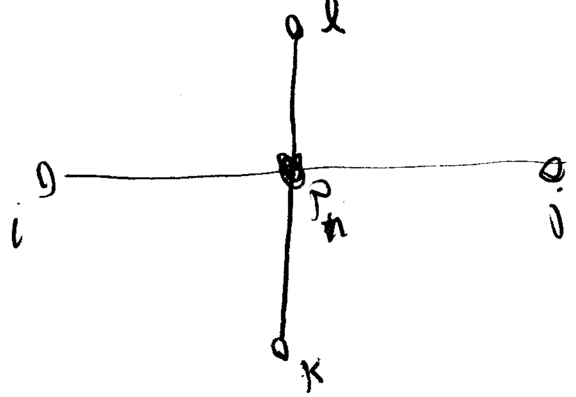
Claim w' equil

Case 1 P_i is an old point

$$\sum_j w_{ij}|P_j-P_i|$$

$$w_{ij}|P_j-P_i| = w_{ij}\frac{|P_j-P_i|}{|P_l-P_i|}(P_l-P_i) = w'_{il}(P_l-P_i)$$

Case 2 P is a new point (add one at a time) of intersect



to show

$$w'_{ni}(P_i - P_n) + w'_{nj}(P_j - P_n) = 0$$

LHT

$$w_{ij}\frac{|P_i - P_j|}{|P_i - P_n|}(P_i - P_n) = w_{ij}(P_i - P_j)$$

Thus

$$w_{ij}(P_i - P_j) + w_{ij}(P_j - P_i) = 0$$ QED

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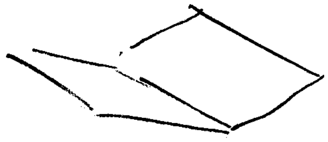
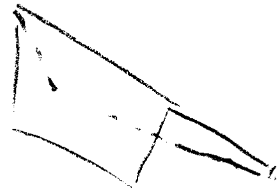
proper (clean)



valley



ridge
mountain

Γ polyedral graph project to plane $G'_A(P')$

<u>Main Argument</u>

outer-zero equilibrium stress $\Leftrightarrow$ outer-flat
Γ is flat outside convex faces

<u>Lemma 5</u> A ridge projects to a bar.

<u>Thm 6</u> $M \subseteq \mathbb{R}^2$ st $p \in M$ is a max value of Γ then
$M = \bigcup$ Face F is exterior to a convex cycle.

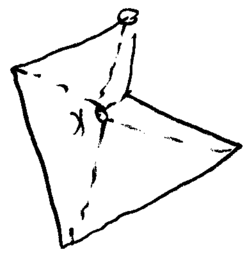
<u>pf</u> consider ∂M

<u>claim</u> edges $\in \partial M$ are the image of a bar
There are cases a to l for ∂M.

<u>Claim</u> only case 1 is possible

<u>Case 4</u> $\partial M =$ vertex x $\quad z =$ height of M

let $X = \text{Project}\{x \in \Gamma \mid z(x) \leq z - \varepsilon\}$



<u>note</u> convex angles are ridges

$\therefore$ example have 4 bars contra!

<u>Claim</u> X has at least 3 bars.