GC

Chernoff Bounds for Traps

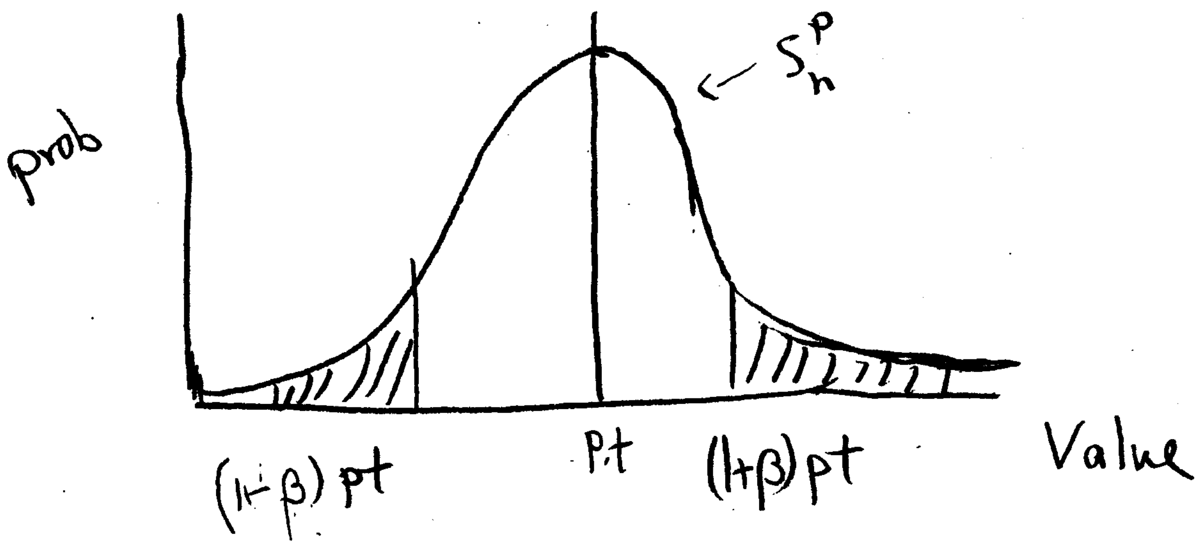
9/18/12

Let $X_1 \cdots X_t$ be independent 0/1 random variables

Assume $\text{Prob}(X_i = 1) = p$

The binomial random variable is

$$S_n^p = X_1 + \cdots + X_t$$



$$\text{Expect}(S_t^p) = \sum E(X_1) = p.t$$

Thm $\text{Prob}\left(S_t^p < (1-\beta)pt\right) < e^{-\beta^2 pt/2} \quad \forall\, 0 \le \beta < 1$

Thm $\text{Prob}\left(S_t^p > (1+\beta)pt\right) < e^{-\beta^2 pt/3} \quad \forall\, 0 \le \beta < 1$

Tail Estimate for Traps

Thm S n-non-crossing line seg, query q, $\lambda > 0$

$$\text{Prob}\left(\text{Search}(q) > 3\lambda \ln(n+1)\right) \le \frac{1}{(n+1)^{\lambda \ln(1.25)-1}}$$

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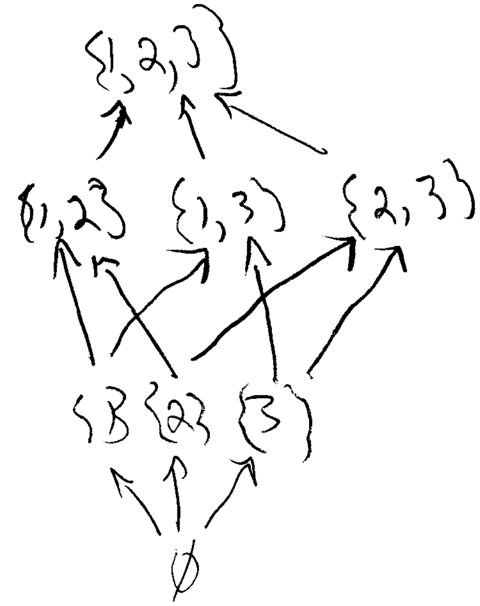
pf Let Z be random variable # nodes on search path.

Goal: Write Z as sum of indep variables.

Consider DAG of all subsets of $\{1,\dots,n\}$



<u>note</u> Each path is an insertion order!

Mark edge if trap containing q changes.

Indegree ≤ 4 by BA

Mark more edges for $i \geq 4$ so indeg is 4.

<u>Def</u> $X_i = \begin{cases} 1 & \text{if } i\text{th edge is marked} \\ 0 & \text{o.w} \end{cases}$

$Z \leq 3Y$ where $Y = \sum X_i$

<u>Goal:</u> bd $\Pr[Y \geq \lambda \ln(n+1)]$

<u>Thm</u> (Markov) $\Pr[X \geq \alpha] \leq E[X]/\alpha \quad X \geq 0$

$E(Y) = \sum E(X_i) = 4H_n$

$E(X_i) \leq 4/i$ $\quad$ (i small ?)

note X_i are ind

<u>Chernoff</u>: $\text{Prob}(Y > (1+\beta)E(Y)) < e^{-\beta^2 \frac{4}{3} H_n}$

$= (e^{H_n})^{-4/3\beta^2}$ $\quad$ wolfram $e^{H_n} \leq n+1$

$\leq \frac{1}{(n+1)^{4/3\beta^2}}$ $\quad$ (Concentration)

<u>Claim</u> At most $2(n+1)^2$ different qs.

pf $2(n+1)$ slabs usin $2(n+1)$ endpoints

each slab has $(n+1)$ traps

<u>note</u> 2 q in each trap have same search!

Prob that some search is more than

$(1+\beta) 4 H_n$ is $\leq \dfrac{1}{(n+1)^{\frac{4}{3}\beta^2 - 2}}$

pick β st $\frac{4}{3}\beta^2 - 2 \geq 1$

$(4/3)\beta^2 \geq 3$

$\beta^2 \geq 9/4$ or $\beta \geq 3/2$