1 Using Cell-Chains

In class we proposed using Cell-Chains to represent topological information. Let \( d \) be the dimension of the representation of a cell complex \( \mathcal{T} \). In this data structure we will maintain the boundary map (the poset) as well as the switch operators \( \alpha_0, \ldots, \alpha_d \).

1. Let \( F \) be a \( d - 1 \)-dimensional cell in \( \mathcal{T} \). Describe the code for updating the data structure to remove \( F \) from \( \mathcal{T} \) and merge the two \( d \)-dimensional faces containing \( F \).

2. Let \( F \) be a \( d \)-dimensional cell in \( \mathcal{T} \). Describe the code for updating the data structure to add a new point \( p \) interior to \( F \). The cell \( F \) should be removed and a new \( d \)-dimensional cell should be added for each \( d - 1 \)-dimensional common to \( F \). These new cells should each contain \( p \).

Hint: Try each of these problems for \( d = 2 \) and \( d = 3 \) first.

2 Triangulating a Trapezoidal Decomposition

Let \( \mathcal{P} \) be a planar straight line graph and \( \mathcal{T} \) a trapezoidal decompositions of \( \mathcal{P} \). Show how to triangulate \( \mathcal{P} \) in \( O(n) \) time using \( \mathcal{T} \).

3 Star Shaped Polygons

A polygon \( \mathcal{P} \) is star shaped if there exists a point in the interior of \( \mathcal{P} \) that can see all of the interior.

1. Give an \( O(n) \) expected time algorithm to determine if a simple polygon of size \( n \) is star shaped.

2. Give a \( O(\log n) \) time algorithm for determining if a point \( q \) is in a star shaped polygon \( \mathcal{P} \). We assume that the the vertices of \( P \) are given in CW order and that we are also given a point \( p \) that can see all of the interior \( P \).

4 Trapezoidal Map for Intersecting Segments

Give an \( O(n \log n + k) \) expected time algorithm for computing a Trapezoidal-Map for a set of \( n \) line segments where the number of segment intersections is \( k \). You may assume that the segments are all in general position and none are vertical.
5 Circular Partition

Given a set of red points \( R \) and a set of green point \( G \) in the plane give an algorithm to find a disk \( D \) such that \( G \subset D \) and \( R \cap D = \emptyset \) if one exists. Your algorithm should run in expected linear time in the size of \( R \) and \( G \).