

CG
11/2/10

Quadtree Via Z-ordering

Input: $P \subseteq [0, 1)^2$

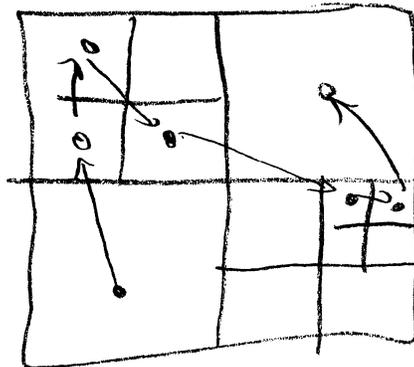
Output: Compressed Quad tree of P .

A Z-order (N-order)

Search (B : Box, $P \subseteq \mathbb{R}^2$)

Search $B_{LL}, B_{UL}, B_{LR}, B_{UR}$

eg



$p \in [0, 1)^2$ consider bit expansion

$$P_x = .x_1 x_2 \dots x_i \dots$$

$$P_y = .y_1 y_2 \dots$$

$$Sh(P_x, P_y) = .x_1 y_1 x_2 y_2 x_3 \dots = Sh(p)$$

Goal: Sorting $Sh(p_1) \dots Sh(p_n)$ is N-order

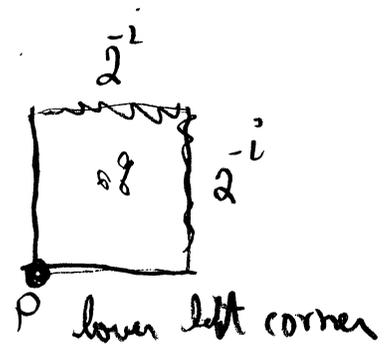
What is a box?

$$P_x = .x_1 \dots x_i$$

$$P_y = .y_1 \dots y_i$$

$$P_x = .x_1 \dots x_i * \dots *$$

$$P_y = .y_1 \dots y_i * \dots *$$



Def Derived Sq. of $P \neq Q$

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Def, Smallest box containing $P \& Q$.

$$Sh(P) = (x_1, y_1, x_2, y_2) \text{ ---}$$

$$Sh(Q) = (x'_1, y'_1, x'_2, y'_2) \text{ ---}$$

def $diff(P, Q) =$ first index where $Sh(P), Sh(Q)$ differ.

def $P \uparrow_i = (x_1 \dots x_i, y_1 \dots y_i)$

$$B(P, Q) = (P \uparrow_{i-1} = Q \uparrow_{i-1}, 2^{-i})$$

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Lemma 1 $P \cap B$ form contiguous set in $\text{Sort}(\text{Sh}(P))$.

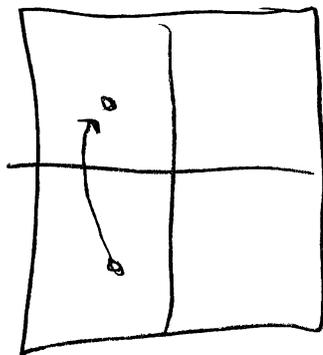
$$B \equiv \begin{pmatrix} P_x = x_1, \dots, x_i, 0, \dots \\ P_y = y_1, \dots, y_i, 0, \dots \end{pmatrix}$$

$q \notin B$ then $\text{dist}(q, P) \leq i$

$\therefore \text{Sh}(q)$ before or after points in box.

Lemma 2 Suppose B is a box subboxes B_1, \dots, B_4
at least 2 B_i are non empty then
two adj points have B as their derived sq.

es

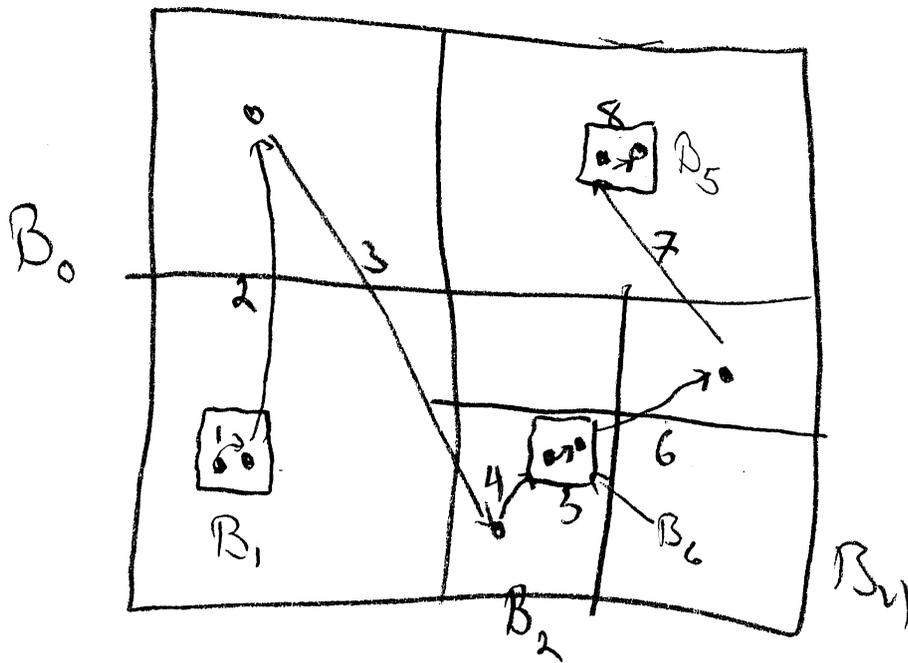


pt Consider adj pair in different subboxes.

Consider derived boxes from adj points.

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es



$$e_1 \equiv B_1$$

$$e_4 = B_2$$

$$e_7 = B$$

$$e_2 \equiv B$$

$$e_5 = B_3$$

$$e_3 \equiv B$$

B

B_4

/

B_2

/

B_6

