Converting a Cycle

Problem: input: 2D polygon $P$

output: A continuous motion of $P$ s.t.
1) $P$ eventually convex
2) $P$ never intersects itself

Answer: Yes

Cor: A simple path can be straightened

Question: Tree Flattening

$\rightarrow$

Answer: No.
Known examples of locked linkages.

3D arc [CJ98]
3D unknotted cycle [CJ98]

3D unknotted cycle [BDD+01]
3D unknotted cycle [Tou01]

2D tree [BDD+02]
2D tree [ICD02]

Degree 8
Degree 3

UNLOCKED LINKAGES
method that can get stuck

pull on ends of a chain
blow air into a cycle

Moves considered by paper.

expansive

\( \forall v, w \in V \) \( \text{dist}(v, w) \) is non-decreasing

strictly expansive

\( (v, w) \neq E \) \( \text{dist}(v, w) \) is increasing

extra-convex configuration

1) each arc not contained in cycle is straight
2) cycle

\[ \text{convex} \]
Thm 1. A - a cycle $A \rightarrow \alpha$ seq of expansion moves to an outer-convex config.

\underline{Basics}

- Linkage in $G = (V, E)$ simple if edge are called basis linkages
- $V \rightarrow p_1, p_2, \ldots, p_n \in \mathbb{R}^2$
- $st \ PGLG$

\underline{Expansiveness}

The Expansiveness on vertices is also edges.

\underline{Case: Vertex & edge point}

\[ \begin{array}{c}
\text{\includegraphics{diagram.png}}
\end{array} \]
\[ B(p, q) = \text{antip} \land p \land q \in \mathcal{B} \]

Claim: \( B(c, p) \subseteq B(p, p) \cup B(p_2, p) \)

Core

\[ p \quad c \quad p_2 \]

These lengths are non-decreasing

\[ p_3 \quad d \quad p_1 \]

Non-decreasing

Reductions

Mod 1: Remove straight vertices

Mod 2: Rigidity convex polygons

Mod 3: Remove components nested within convex cycles.
Mod 4 Add struts (all pairs) (triangles?)

Model now has bars & struts

Mod Find $G_a(P) \rightarrow G'_a(P)$

Reduce & bag-struts
1) add intersection
2) remove parallel struts

Def. Link expansion if $A(ij) \in$ Bars

- dist($ij$) unchanged
- $A(ij) \in$ Struts
- dist($ij$) non-decreasing

Then link expansion $\Rightarrow$ expansion in $G_a(P)$
Infinitesimal Motion

\[ V = (V_i - V_n) \quad V \in \mathbb{R}^2 \]

\[ V = 0 \]

\[ V_{ij} = (\mathbf{p}_j - \mathbf{p}_i) \text{ change } (\mathbf{y} - \mathbf{y}_i)(\mathbf{p}_j - \mathbf{p}_i) \text{ in general} \]

Goal: \noindent \underline{\text{vst}}

\[ (V_{ij} - V_i) \cdot (\mathbf{p}_j - \mathbf{p}_i) = 0 \quad (i,j) \in \mathbb{D} \]

\[ \geq \quad \text{ES} \]

(2)
Equilibrium Stresses

\[ \text{stress} W! \text{ Links } \rightarrow R \quad w_{ij} \quad \text{force} \]

\[ w_{ij} < 0 \quad \text{then edge is pushing} \]

\[ w_{ij} > 0 \quad \text{pulling} \]

\[
\sum_{j} w_{ij} (p_j - p_i) = 0
\]

\[
\text{W is proper} \quad w_{ij} < 0 \quad (i,j) \in S
\]

Lemma 3: If only proper equal stress is zero, then 3 infinitesimal motion satisfying 12.

\[ \text{88 dual LP} \]
Definition: A stress is outer-ger if

\[ w_{ij} \neq 0 \iff (i,j) \text{ on convex cycle} \]

interior \& compact

Lemma: Wi imposes proper equal stress on \( G_k(P) \) and \( G_k'(P') \)

the \( j \text{th } \alpha \frac{\text{outer-magn}}{\text{inner-magn}} \)

\[ P_k \]

\[ P_k, P_i, P_j \]

\[ W_{kl} = W_{ij} \frac{\| P_i - P_j \|}{\| P_k - P_i \|} \]
Maxwell-Cremona Theorem

Plane: bow-strut $G = (V, E)$

lift $G$ to 3D at faces as co-planar

modify extremum face

Sudia $F$

$F = \mathbf{3}(P) = \mathbf{a} \cdot \mathbf{P} + \mathbf{b}$ a gradient

$F' = \mathbf{3}(P) = \mathbf{a}' \cdot \mathbf{P} + \mathbf{b}'$ a 'stress'

$\mathbf{d} \mathbf{G}_{ij}^+ \in \mathbb{R}^2$

perpendicular to $(i, j)$ in $\mathbb{R}^2$

$\| \mathbf{e}_{ij}^+ \| = \| \mathbf{P}_i - \mathbf{P}_j \|

\# \mathbf{a} - \mathbf{a}' = \mathbf{W}_{ij} \mathbf{e}_{ij}^+$ stress
\[ \text{Thm (Maxwell-Cremona)} \]

(i) \( W \) as defined by \( F \) is an equilibrium stress of \( G(p) \)

(ii) \( W \) is an equal stress on plane \( G(p) \)

Then \( F \) is equal.