Representing Topological Information

Planar Subdivision = Partition of the plane into

Vertices
Edges
Faces

Vertices are closed
Edges & Faces are open
Face = maximally connected region

Degeneracies not allowed
1) Isolated vertex on a face on edges

Types of Topological Info

Boundary info:

Poset
Need
1) order of edges at a vertex
2) edges around a face.

2D solution
1) doubly-connected edge list
   a) write each edge as two arcs

   next (arc); arc
   prev (arc); arc

   reflection (arc) = twin
Group Theoretic Approach

Next, prev, reflect are permutation

$A = \text{set of arcs}$

$q \in \text{next then } q \in \text{Sym}(A)$

$q^{-1} = \text{prev}$

$r = \text{reflect} \quad r \in \text{Sym}(A)$

proof of $R$ fixed-point-free for

$R^2 = \text{id}$

Set $\tau_1, \ldots, \tau_k \in \text{Group} \quad \langle \tau_1, \ldots, \tau_k \rangle = \text{subgroup generated by } \tau_1, \ldots, \tau_k$

Orbits $\langle \tau_1, \ldots, \tau_k \rangle$

$a \sim b \iff \exists r \in \langle \tau_1, \ldots, \tau_k \rangle \text{ such that } r(a) = b$

Orbits $\langle (R) \rangle = \text{edges}$

Orbits $\langle \langle q \rangle \rangle = \text{faces}$

Orbits $\langle \psi R \rangle = \text{edges}$

Orbits $\langle SR, q \rangle = \text{connect components}$
View \#1, \#2, acting triangles.

1) add point in "middle" of each face and form triangles

\[ \begin{array}{c}
\text{Note 1-1 correspondence} \\
\text{arcs & new triangles} \\
\text{\# 1 takes tri to tri}
\end{array} \]

Another view: Glueing rules:

1) Start with \# triangles = \# arcs

2) Glue triangles with rules \((\#, \#)\)
Bare Centric Subdivision
This construction will work in any dim
1) add a new point in each cell (label by the dim of cell)
2) form simplices out of new points

Switch operators $\alpha_0, \alpha_1, \alpha_2$ (Reflections)

Properties

$\alpha_i^2 = id$

$\alpha_0 \alpha_2 = \alpha_2 \alpha_0$
Return to Posets

Assume:
1) no pinch edge
2) no pinched face
3) face boundary is connected

Claim 1-1 correspondence between numbered trees & paths on the Poset

Def: cell-tuple \((V, E, F)\)
$\alpha, \beta, \gamma$ act on cell-triples

$\alpha_0$ depends on the triple $\left( T, V, E \right)$

$\alpha_0 \left( T, V, E \right) = V'$
Claim: if we drop 1)

4 numbered Fri

2 tuples

Solution: add multiple edges

def path are cell ch

4 cell-chains

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