CG
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# Smallest Enclosing Disc

Application: robot arm placement

Input: $P \subseteq \mathbb{R}^2$ finite
Output: Disk D st $P \subseteq D$ & D has min radius

$P = \{P_1 \cdots P_n\}$ $P_i$ in random order

$\partial D_i \equiv$ boundary

$P_i = \{P_1 \cdots P_i\}$ $D_i \equiv$ min enclosing disk.

Lemma 1) $P_i \in D_{i-1} \Rightarrow D_i = D_{i-1}$

2) $P_i \notin D_{i-1} \Rightarrow P_i \in \partial D_i$

Alg MiniDisc (P)

1) $D_2 = \text{Disc}(P_1, P_2)$

2) for $i \leftarrow 3$ to n

do if $P_i \in D_{i-1}$ then $D_i = D_{i-1}$

else $D_i = \text{MiniDiscWithPoint}(\{P_1 \cdots P_{i-1}\}, P_i)$

return $D_n$

Mini Disc with Point $(P, q)$

Output: Mini Disc with $q$ on $\partial$.

1) randompermute $(P)$

2) $D_1 = \text{Disc}(q, p_1)$

3 for $j \leftarrow 2$ to $n$

do if $p_j \in D_{j-1}$ then $D_j \leftarrow D_{j-1}$

else $D_j = \text{MiniDiscWith2Points}(\{p_1 \cdots p_{j-1}\}, p_j, q)$

Mini Disc with 2 Points

1 $D_0 = \text{Disc}(q_1, q_2)$

for $k \leftarrow 1$ to $n$

do if $p_k \in D_{k-1}$ then $D_k \leftarrow D_{k-1}$

else $D_k \leftarrow \text{Disc}(q_1, q_2, p_k)$

Timing

1) Mini Disc 2 Points is $O(n)$ time

$n$ incircle tests

2) Mini Disc 1 Point $(P, q)$

backwards analysis

Def $|P_i| = i$ $\quad P_i \subseteq P$ $\quad p \in P_i$ is critical if $p \in \partial D_i$

$$|\partial D_i| \leq 3$$

cost $p$ not critical $\quad c$

critical $\quad c \cdot i$

Sum of possible removals $\quad 2c \cdot i + c(i-2) \leq 3ci$

expected cost $\leq 3ci/i = 3c$

Mini Disc Point is $O(n)$ expected

Mini Disc

random variable $Y_i = \begin{cases} 1 & \text{if outside} \\ 0 & \text{o.w} \end{cases}$

$P_r$

$X_i = \text{cost minidiscOnePoint}(P_i, q)$

$Z_i = C + X_i \cdot Y_i$

$E(Z_i) = E(C + X_i Y_i) = C + E(X_i \cdot Y_i)$

$+ E(X_i) \cdot E(Y_i)$

$Ci \left(\frac{2}{i}\right) = 3C$