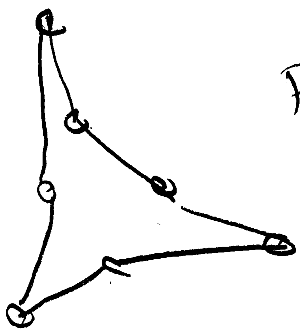
CG
11/18/08

Pseudo-Tri & Rigidity

Pseudo-tri examples

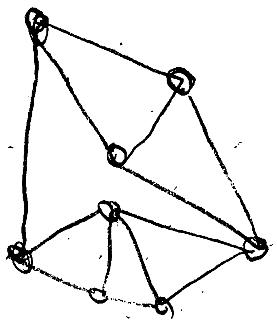


Pseudo-Δ

Vertex types
convex - corners
straight
reflex

inside is P-3-gon
outside is P-4-gon

Pseudo-triangulation (including CH) P-Δ



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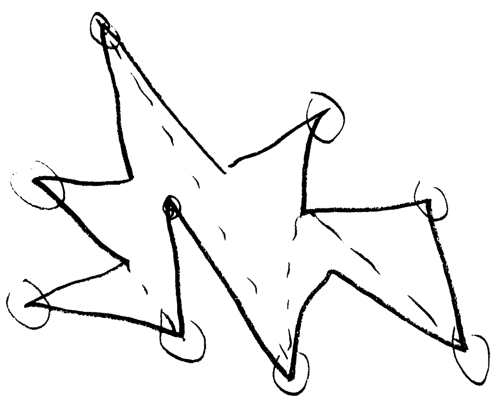
triangulating a Polygon



A point is <u>pointed</u> if some angle is reflex.

G is <u>pointed</u> if all $v \in V$ are pointed.

<u>Def</u> G is <u>Infinitesimal rigid</u> if

$\forall V = (v_1, \cdots, v_n)$, $v_i \in \mathbb{R}^2$ if

$$(v_i - v_j) \cdot (P_i - P_j) = 0 \quad \forall i,j \in E$$

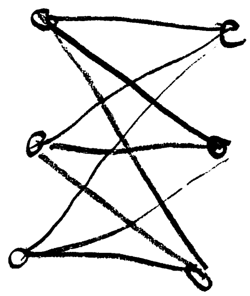
implies $V \equiv 0$

<u>Def</u> G is <u>minimally rigid</u> if
G is Inf-rigid & $G \setminus e$ is not $\forall e \in E(G)$

<u>Def</u> G is Laman if $m = 2n - 3$
$\forall V' \subseteq V$ if $k = |V'|$ then G induced on V' has $\leq 2k-3$ edges

Thm G is minimally rigid iff G is Laman.

eg $K_{3,3}$



$n = 6$

$m = 9 = 2 \cdot 6 - 3$

Question: Minimally rigid planar graphs?

Thm Pointed PΔ are Minimally rigid

Thm G is a Pointed PΔ & $e \in CH(G)$

then expanding e gives an expansive motion.

Convexifying a Polygon

Alg (R polygon)

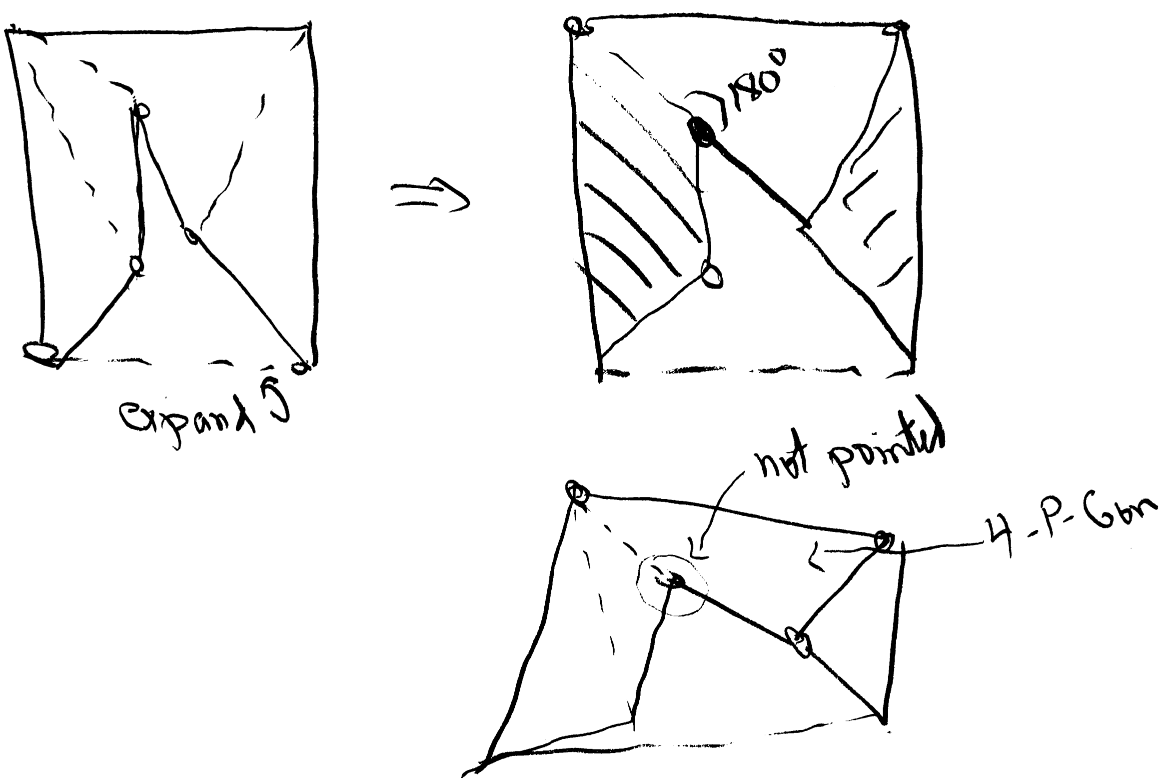
1) Add struts to R forming pointed PΔ (convex)

2) While ∃ strut s on CH do

a) expand s until some pair e & e' are straight

i) if e & e' ∈ Bars then freeze e & e'

else clean up by edge flipping



<u>Lemma</u> If T is P-Δ of n points P then

$$m = |E(T)| = 2n-3$$

$t \equiv \#\Delta$ in T $\quad m = \#$edges

Euler: $t - m + n = 1$ (no outside face)

$\#$angles $= 2m$

$\#$ corners $\equiv 2m - n$ (pointed condition)

$$3t \equiv 2m - n$$

Euler $3t - 3m + 3n = 3$

$$2m - n - 3m + 3n = 3$$

$$-m + 2n = 3$$

$$m = 2n - 3$$

<u>Claim</u> T embedded graph $m < 2n-3$ then T is not rigid!

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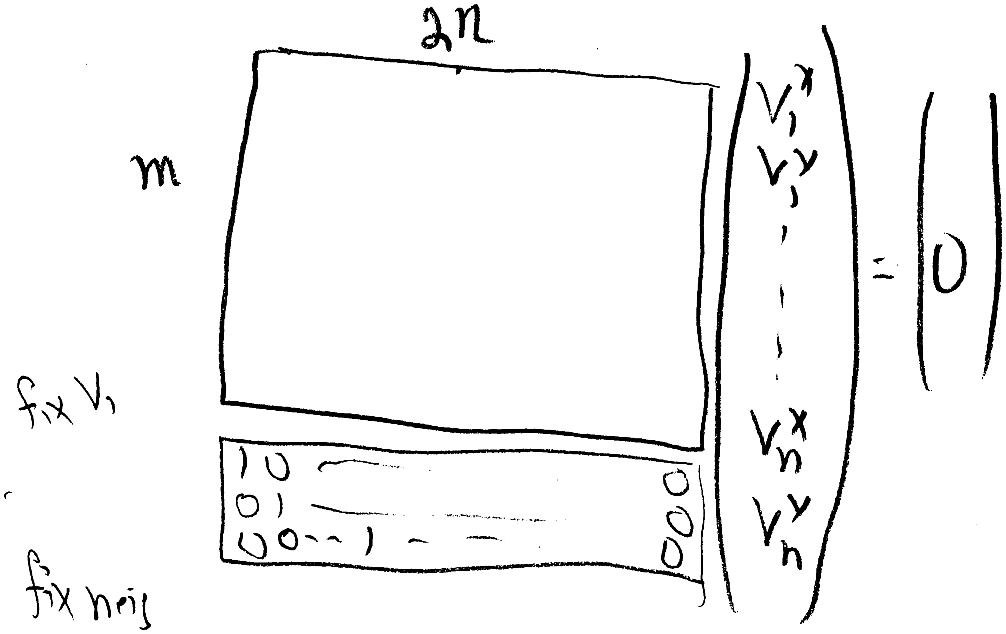
ie $\exists\, v = (v_1, \dots, v_n)$ not a rigid motion

$$(v_i - v_j) \cdot (p_i - p_j) = 0 \quad \forall (i,j) \in E$$



$m+3 < 2n$ done!

Thm T is pointed PΔ of a CH then T is rigid.

We consider dual problem!

Maxwell lifting

Claim If f is a lifting of a face F & P is a reflex vertex of F then $P \in \operatorname{Max}_f(F)$ iff f on F is constant.