Convexifying a Cycle

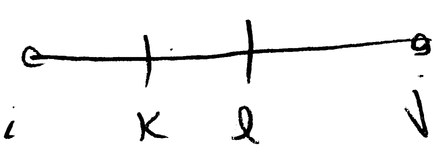
<u>Def</u> A <u>stress</u> is <u>outer-zero</u> if

$w_{ij} \neq 0 \Rightarrow e_{ij} \in$ Convex Cycle or inter to Con Cycle

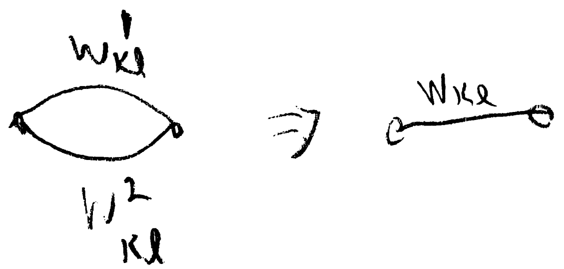
<u>Def</u> A stress $w_{ij} \neq 0$ $e_{ij} \notin CC$ is called outer-nonzero.

<u>Lemma</u> $\exists w$ of $G_A(P)$ w in nonzero proper equilibrium then $\exists w'$ of $G'_A(P')$ outer-nonzero proper equil stress.

<u>Pf</u> strut



$$w'_{kl} = w_{ij} \frac{|P_i - P_j|}{|P_k - P_l|}$$



$$w_{kl} = w^1_{kl} + w^2_{kl}$$

<u>Claim</u> w' equil

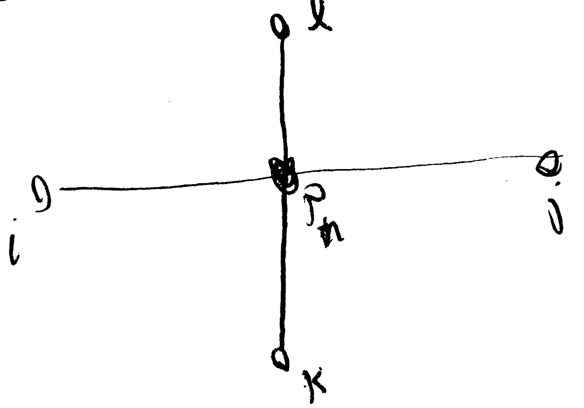
<u>Case 1</u> P_i is an old point

$$\sum_j w_{ij} |P_j - P_i|$$

$$w_{ij}|P_j - P_i| = w_{ij} \frac{|P_i - P_j|}{|P_l - P_i|}(P_l - P_i)$$

$$= w'_{il}(P_l - P_i)$$

Case 2 P is a new point (add one at a time) pt intersect



$$w'_{ni}\,\underline{(P_i - P_n)} + w'_{nj}\,|P_j - P_n| \stackrel{?}{=} 0$$

$$w_{ij}\frac{|P_i - P_j|}{|P_i - P_n|}(P_i - P_n) + w_{ij}$$

$$w_{ij}(P_i - P_j) + w_{ij}(P_j - P_i) = 0$$

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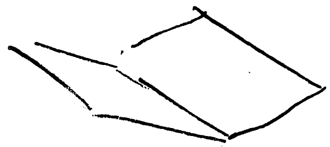
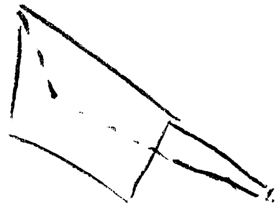
proper (clean)



valley



ridge
mountain

Γ polyedral graph project to planar $G'_A(P')$

<u>Main Argument</u>

outer-zero equilibrium stress $\Leftrightarrow$ outer-flat
Γ is flat outside convex faces

<u>Lemma 5</u> ridge project to a bar.

<u>Thm 6</u> $M \subseteq \mathbb{R}^2$ st $p \in M$ is a max value of Γ.

$M = \bigcup_{\forall F} \text{Face}$ F is exterior to an convex cycle.

<u>pf</u> consider ∂M

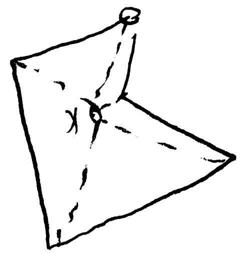
<u>claim</u> edges $\in \partial M$ are the image of a bar

there are a to l case for ∂M

Claim only case 1 is possible

Case 4 $\quad \partial M = \text{vertex } x \quad z = \text{height of } M$

let $X = \text{Project } \{x \in \Gamma \mid z(x) \leq z - \varepsilon\}$



note convex angles are ridges

$\therefore$ example have 4 bars contra!

Claim X has at least 3 bars