CG
10/29/08

Lower Bound on Mesh Size

Suppose M is an optimal size mesh for PSLG G with Bounding Box B.

No angle $< \alpha$.

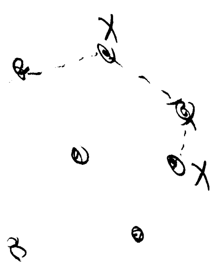
Question: Bd $|M|$ from below.

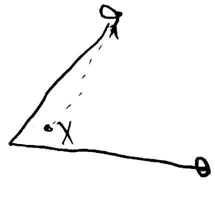
Feature $\equiv$ vertex, edge, face

$f, g \in$ Feature then f & g are disjoint if $f \cap g = \emptyset$

Def (Local feature Size) $x \in \mathbb{R}^2$ then

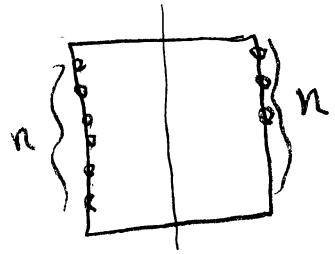
$lfs(x)$ = distance from x to 2nd nearest feature (G PSLG)





Goal: (Thm) $|M| = \Theta\left(\frac{1}{\alpha}\int_{x\in B} 1/\mathrm{lfs}^2(x)\right)$

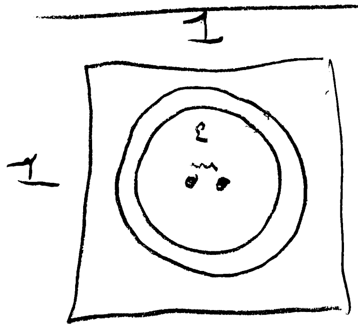
eg:



$= \mathrm{PSLG}$

$$\int_{x\in B}\frac{1}{\mathrm{lfs}^2(x)} = 2\int_{x\in B/2}\frac{1}{\mathrm{lfs}^2(x)} \approx 2\int_0^1\int_0^{1/2}\frac{1}{(x+1/n)^2}\,dx\,dy \approx 2\int_{1/n}^{1/2}\frac{1}{x^2}\,dx$$

$$= 2\Big[1/x\Big]_{1/2}^{1/n} = 2(n-2) \approx 2n$$



$$\int_\varepsilon^1 \frac{2\pi r}{r^2}\,dr = 2\pi\log r\Big]_\varepsilon^1 \approx -\log\varepsilon = \log 1/\varepsilon$$

Thm $E \cap F = X$ in M Then $|E|/|F| \leq 2^{1/\alpha}$

See Thm 1 in handout

Suppose V_x is a Voronoi cell.

Def R = radius of small ball containing V_x centered at x.
r = " "largest" "contained in V_x" "

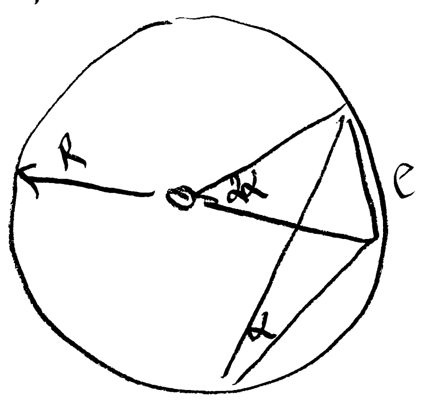
Aspect-Ratio of $V_x = R/r$

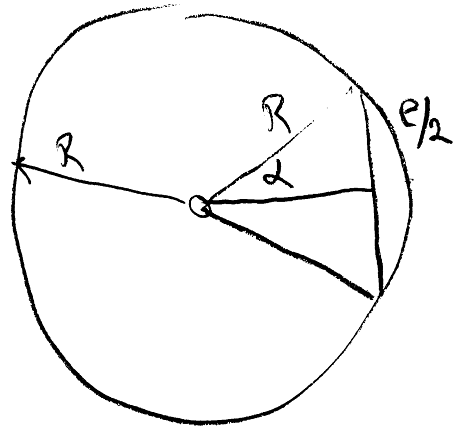
Note: $2r$ = distance to $NN(x)$.

Cor Aspect-Ratio of $V_x \leq \left(\frac{1}{2\sin\alpha}\right) 2^{1/\alpha}$

Lemma Δ with min angle α Then Radius-edge$(\Delta) \leq \frac{1}{2\sin\alpha}$

pf





Consider case $G \equiv P$ (finite set of n points)

<u>Lemma</u> Let $P \subseteq T$ & V_x Voronoi cell of $T \wedge z \in V_x$

$lfs_T(z) \geq \max(dist(z,x), r)$

pf

Let D = lfs disk of z

$z \in V_x \Rightarrow x \in D \Rightarrow lfs(z) \geq dist(z,x)$

suppose $dist(z,x) \leq r$

$dist(z,x) + dist(z,y) \geq dist(x,y) \geq 2r$

$\Rightarrow dist(z,y) \geq r$

Lemma $\displaystyle\int_{v \in V_x} \frac{1}{lfs^2(v)}\, dA = O(1/\alpha)$

$$\int_{v \in V_x} \frac{1}{lfs^2(v)}\, dA \le 2\pi \int_{t=0}^{R} \frac{r\, dt}{(\max(t, r))^2}$$

$$= 2\pi \int_{0}^{r} \frac{t\, dt}{r^2} + \int_{r}^{R} \frac{t\, dt}{t^2}$$

$$\pi + \log(R/r)$$

Thm $\displaystyle |M| = \Omega\left(\alpha \int_B 1/lfs^2\right)$

Handling Edges

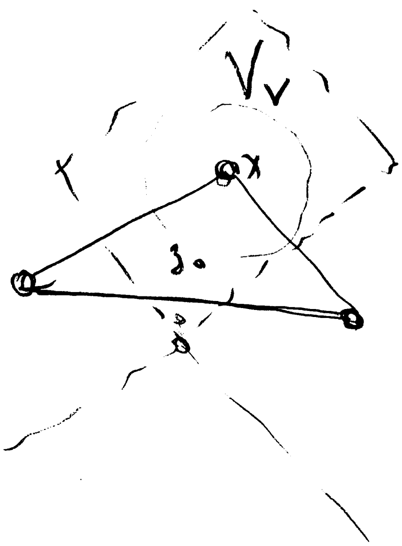
<u>Lemma</u> Δ with vertex V & edge E.

$$\text{dist}(V, E) \geq \frac{r}{2} \cos\alpha.$$

<u>Lemma</u> $G \equiv$ PSLG & $\text{lfs} = \text{lfs}_G$

$z \in V_x$ then $\text{lfs}(z) \geq \max(\text{dist}(z, x), r\cos\alpha)$

<u>Pf</u> <u>prob</u>



<u>Lemma</u> $\displaystyle\int_{z \in V_x} \frac{dA}{\mathrm{lfs}_\sigma^2(x)} \leq O(1/\alpha)$

<u>Thm</u> $\displaystyle |M| = \Omega\left(\alpha \int_B \frac{1}{\mathrm{lfs}^2_\sigma}\right)$