B-splines

Make curves out of multiple Bezier curves

degree \( R = 2 \)

2 control polygons \((a_1, a_2, a_3) \& (b_1, b_2, b_3)\)

\[ a_2 = b_1 \]

Our curve is connected

what is velocity at \( a_3 = b_1 \)?

left: \[ 2(a_3 - a_2) \]

right: \[ 2(b_3 - b_2) \]

\( a_2, a_3 = b_1 \), \( b_2 \) nonlinear than smooth

in general \( a_3 - a_2 \neq b_3 - b_2 \)

we may have a speed bump! why?
Solution

Trace $C_a$ at different rate from $C_b$

\[ \text{pick n st } (n - 0, k(a_3 - a_1)) = \frac{1}{2-n}(b_2 - b_1) \]

Let's think of point $a_3 = b_2$ determined by $(0, n, 2)$

- knot seq $(0, n, 2) = (u_0, u_1, u_2)$
- $d_0 = a_1$
- control points $d_0, d_1, d_2, d_3$
- $d_1 = a_2$
- $d_2 = b_2$
- $d_3 = b_3$

**Algorithm**

\[ C(u) \]

\[ \text{set } d = \left( \frac{u_1 - u_0}{u_2 - u_0} \right) d_2 + \left( \frac{u_2 - u_1}{u_2 - u_0} \right) d_1 \]

if $u_0 \leq u \leq u_1$ then

\[ C(d_0, d_1, d)(u) \]

if $u_1 < u \leq u_2$ then

\[ C(d_1, d_2, d_3)(u) \]
Control points \( d_0, \ldots, d_5 \) de Boor points
Beginpoints \( b_1, \ldots, b_5 \)
Knots \( u_0, \ldots, u_6 \) \( u_i \leq u_{i+1} \) \( u_i \neq u_{i+1} \)

\[
b_i = \left( \frac{u_i - u_{i-1}}{u_{i+n} - u_{i-1}} \right) d_{i+n} + \left( \frac{u_{i+n} - u_i}{u_{i+n} - u_{i-1}} \right) d_i
\]

Note \( b_1 = d_0 \) iff \( u_0 = u_1 \) \& \( b_5 = d_5 \) iff \( u_5 = u_6 \)

Alg for each \( b_i \):
1. \( b_i \rightarrow b_{i+1} \)
2. Make gradient Begin
Bézier Triangle

\( \partial \rightarrow \mathbb{R} \)
\( \partial \rightarrow \mathbb{R}^2 \)

\( P \in \partial \)
\( P = (\alpha, \beta, \gamma), \quad \alpha + \beta + \gamma = 1 \)
\( \alpha, \beta, \gamma \geq 0 \)

\( p \rightarrow \mathbb{R}^3 \)
\( P_i = (\alpha_i, \beta_i, \gamma_i) \in \partial \)
\( P_j \)
\( P_k \)
\( T' = \langle P_j, P_k, P \rangle \)
\( P' \in (\alpha, \beta, \gamma) \in \partial \)