Spectral Rounding

Spectral
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Application Image Segmentation

Input: Image ie pixel (voxel) intensities P_{ij}

Step 1 Make an affinity graph.

$$\boxed{P_{ij}}\ \boxed{P_{i\,j+1}}$$

$$e_{(ij),(i,j+1)} = |P_{ij} - P_{i\,j+1}|^{-1},\ (P_{ij} - P_{i\,j+1})^{-2}$$

$$e^{-(P_{ij} - P_{i\,j+1})^2}$$

Step 2 Find a good edge cut into k pieces

eg

$$NC(G) = \underset{\substack{V_1 \cdots V_k \\ \text{partition}}}{\operatorname{argmin}} \frac{1}{k} \sum_{i=1}^{k} \frac{Cut(V_i, \bar{V}_i)}{Vol(V_i)}$$

Spectral Rounding Heursitic

Suppose λ simple eigenvalue of G

$$Ax = \lambda D x \quad \text{normalized eigen pair}$$

Suppose $e \in G$

Question: How does changing e affect λ

ie $w_t(e) = w(e) + t$

$$\frac{d\lambda}{de} = \lambda_e \qquad A_e, D_e \; X_e$$

<u>Claim</u> $\lambda_e = (x_i - x_j)^2 - \lambda(x_i^2 + x_j^2)$

$$x^T A = \lambda x^T D$$

$$x^T D x = 1$$

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$$\frac{\partial (Ax)}{\partial e} = \frac{\partial (\lambda D x)}{\partial e}$$

$$A_e x + A x_e = \lambda_e D x + \lambda D_e x + \lambda D x_e$$

$$x^T A_e x + x^T A x_e = \lambda_e x^T D x + \lambda x^T D_e x + \lambda x^T D x_e$$

Note: $x^T A x_e = \lambda x^T D x_e$

$x^T D x = 1$

$$x^T A_e x = \lambda_e + \lambda x^T D_e x$$

$$\lambda_e = x^T A_e x - \lambda x^T D_e x$$

$$= (x_i - x_j)^2 - \lambda (x_i^2 + x_j^2)$$

$A_e = E^{os}_{ij}$

$$D_e = \begin{array}{c} \\ i \\ \\ j \end{array} \begin{array}{c} \begin{array}{cc} i & j \end{array} \\ \begin{pmatrix} 1 & \\ & \ddots \\ & & 1 \end{pmatrix} \end{array}$$

Def $\nabla \lambda = \begin{pmatrix} \lambda_{e_1} \\ \vdots \\ \vdots \\ \lambda_{e_m} \end{pmatrix}$

Alg 1) Compute $Ax = \lambda_2 D x$

2) $E^* = E - \delta \nabla \lambda_2$ some "small" δ

2') zero out neg weight edges

3) repeat while $\lambda_2 \neq 0$.