

# **A Rehabilitation of Message-Passing Concurrency**

Frank Pfenning  
Carnegie Mellon University

PWLConf 2018, St. Louis

# A Paper I Love

Types for Dyadic Interaction\*

Kohei Honda

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Department of Computer Science, Keio University

3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223, Japan

## Abstract

We formulate a typed formalism for concurrency where types denote freely composable structure of dyadic interaction in the symmetric scheme. The resulting calculus is a typed reconstruction of name passing process calculi. Systems with both the explicit and implicit typing disciplines, where types form a simple hierarchy of types, are presented, which are proved to be in accordance with each other. A typed variant of bisimilarity is formulated and it is shown that typed  $\beta$ -equality has a clean embedding in the bisimilarity. Name reference structure induced by the simple hierarchy of types is studied, which fully characterises the typable terms in the set of untyped terms. It turns out that the name reference structure results in the deadlock-free property for a subset of terms with a certain regular structure, showing behavioural significance of the simple type discipline.

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## Types for Dyadic Interaction\*

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- *Types for Dyadic Interaction*, Kohei Honda, CONCUR 1993
- With some newer developments
  - *Session Types as Intuitionistic Linear Propositions*, Luís Caires & Pf., CONCUR 2010
  - *Manifest Sharing with Session Types*, Stephanie Balzer & Pf., ICFP 2017

Types for Dyadic Interaction\*

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sort(A);  
x = A[0];
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hd(sort(A))
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  - Operationally (how)

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# The Activity of Programming

- Every programmer, all the time, reasons

- Operationally (how)

- Logically (what)

- The effectiveness of a programming language depends critically on

- How programs execute

- What they achieve

- Which reasoning principles connect the operational and logical meaning of a program

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# Why is Functional Programming So Effective?

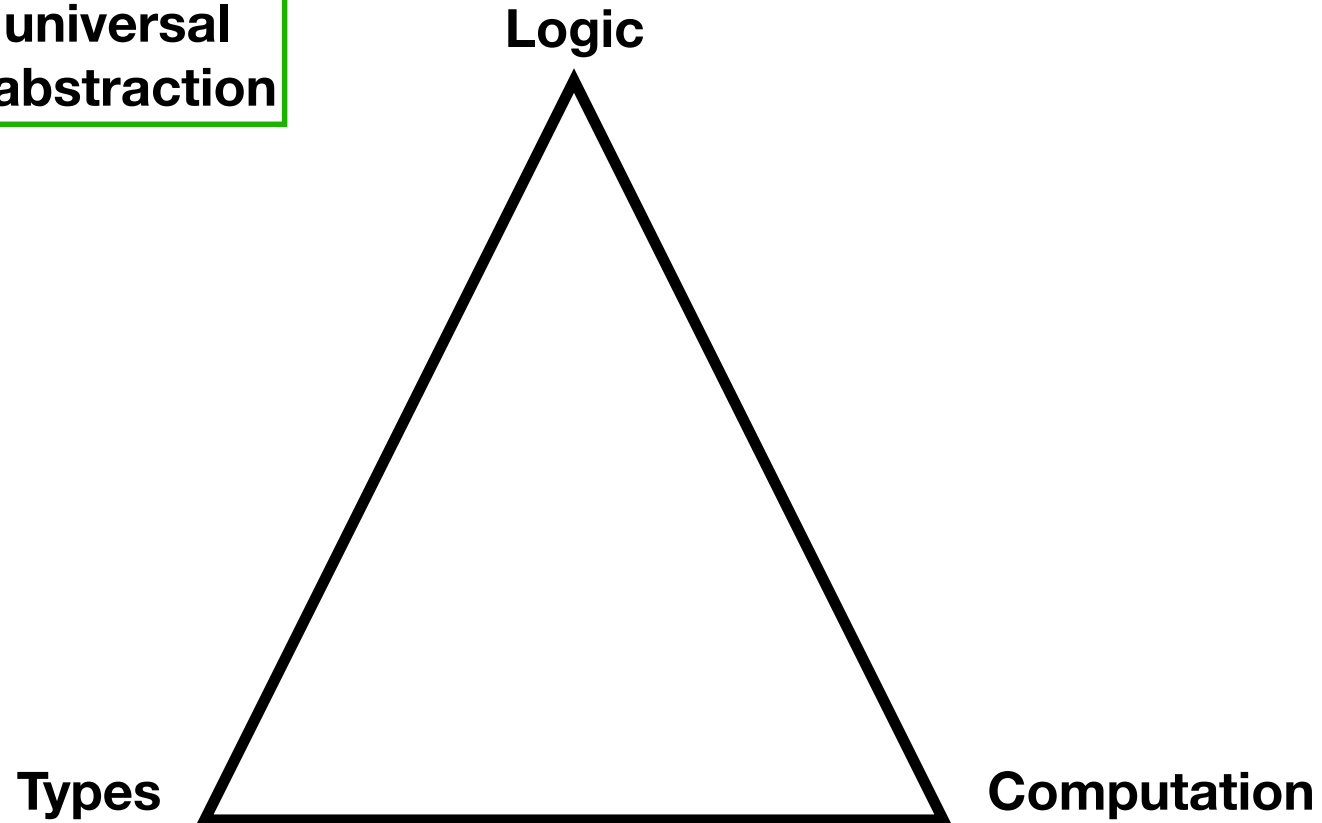
# Why is Functional Programming So Effective?

**Functions are a universal  
and fundamental abstraction**



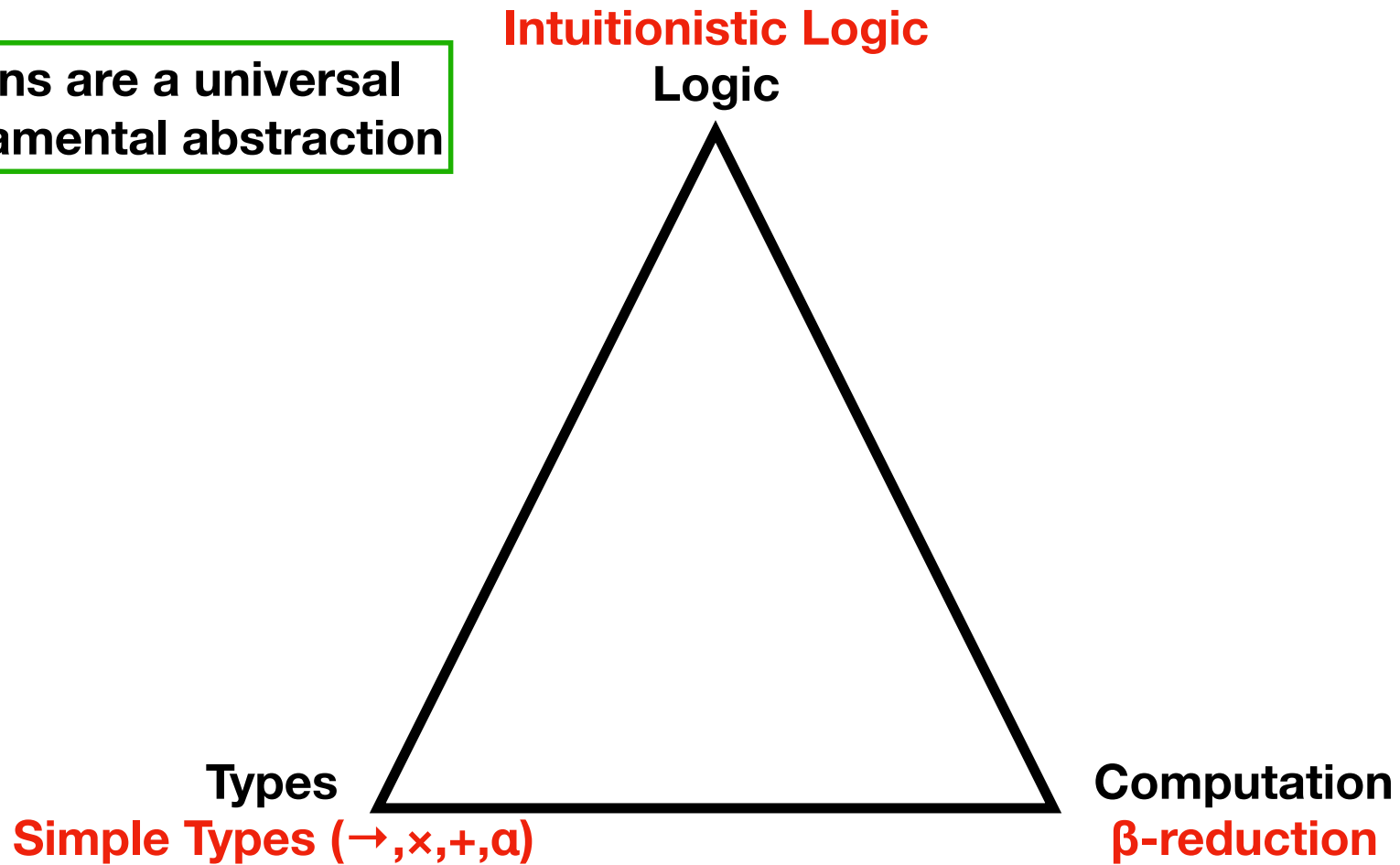
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Intuitionistic Logic

Logic

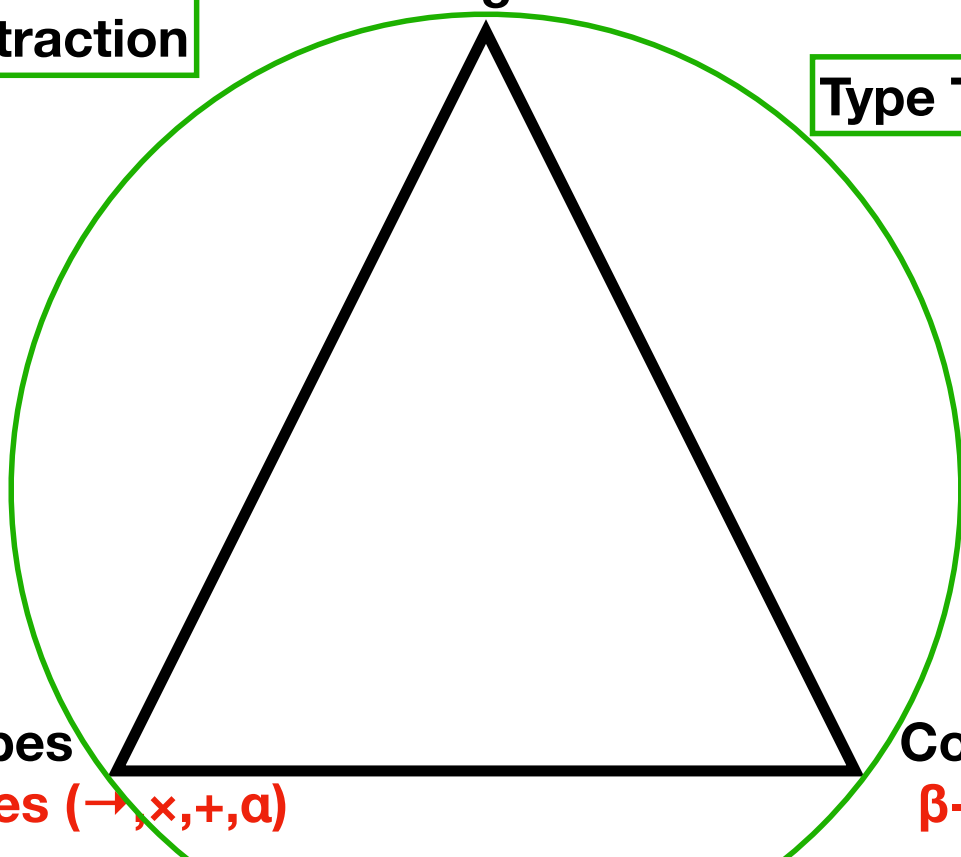
Type Theory

Types

Simple Types ( $\rightarrow, \times, +, \alpha$ )

Computation

$\beta$ -reduction



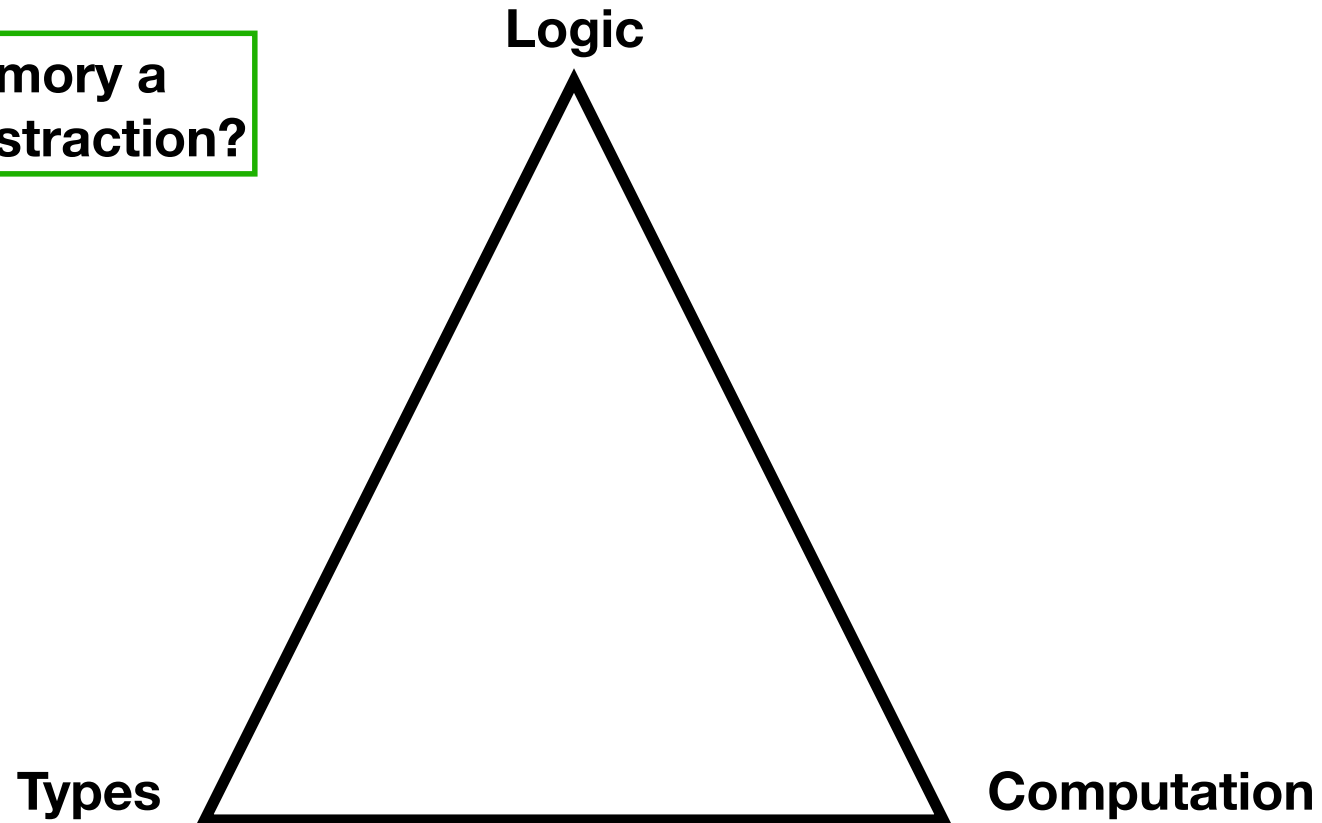
**What about Concurrency?**

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Concurrent Separation Logic

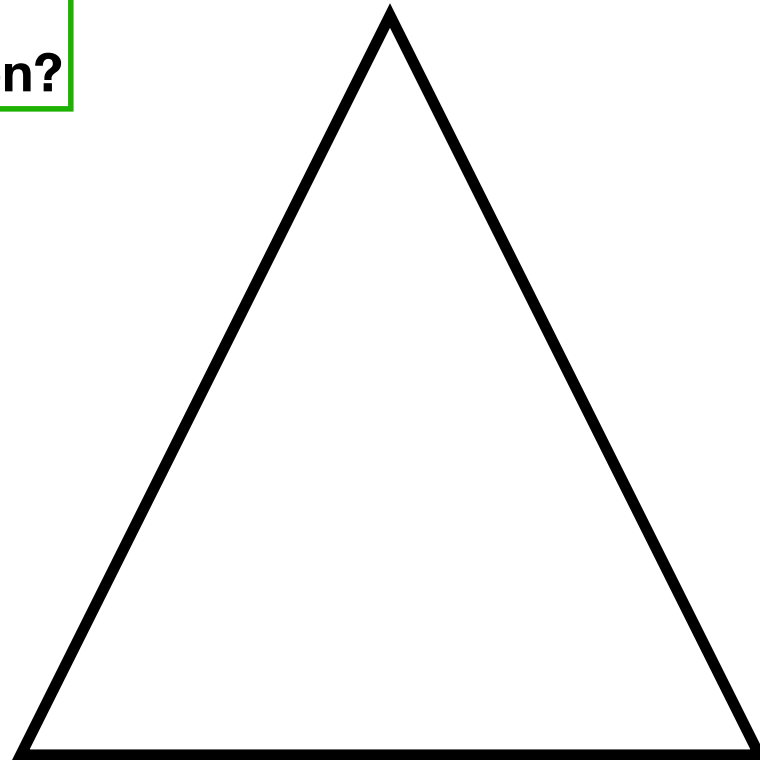
Logic

Is Shared Memory a  
Fundamental Abstraction?

Types  
?

Computation

Read/Write Shared Memory



# What about Concurrency?

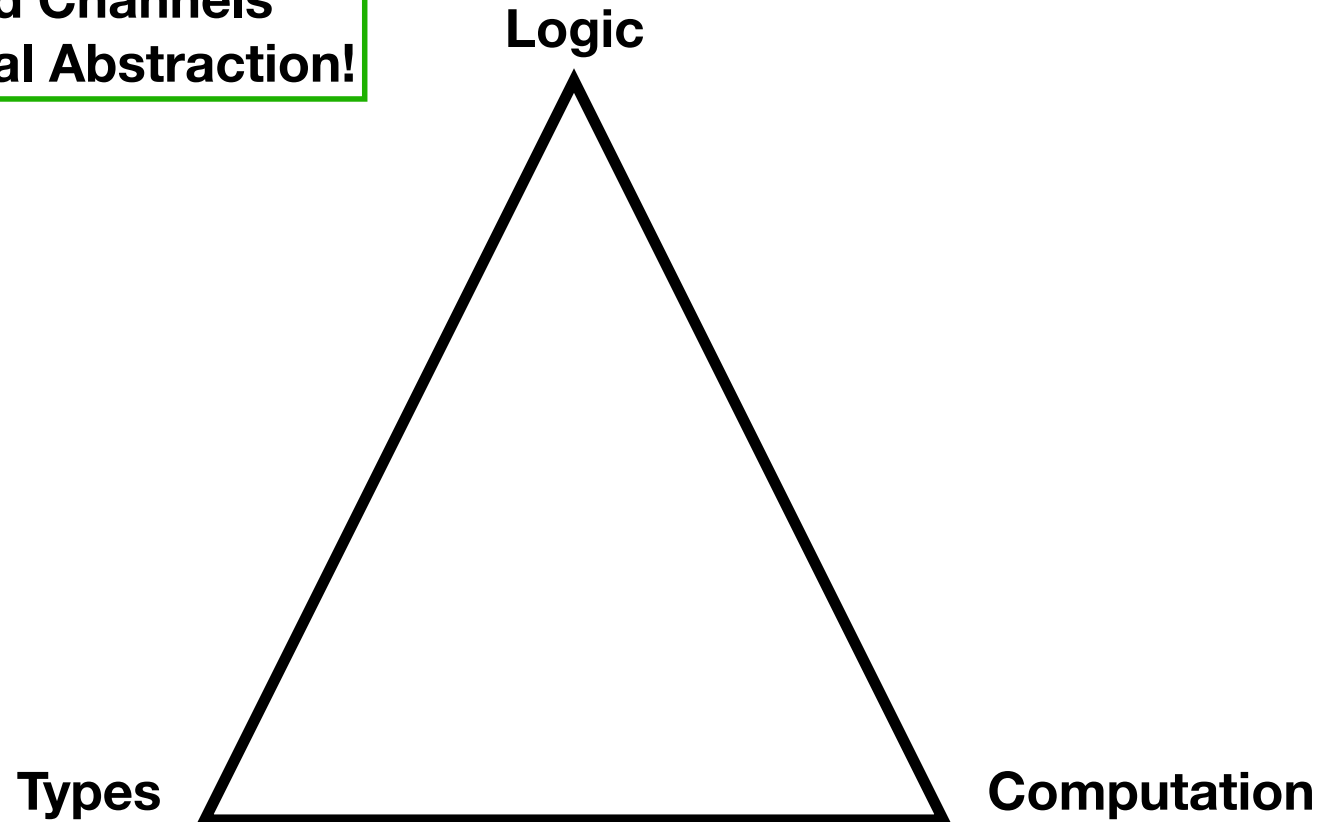


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**Processes and Channels  
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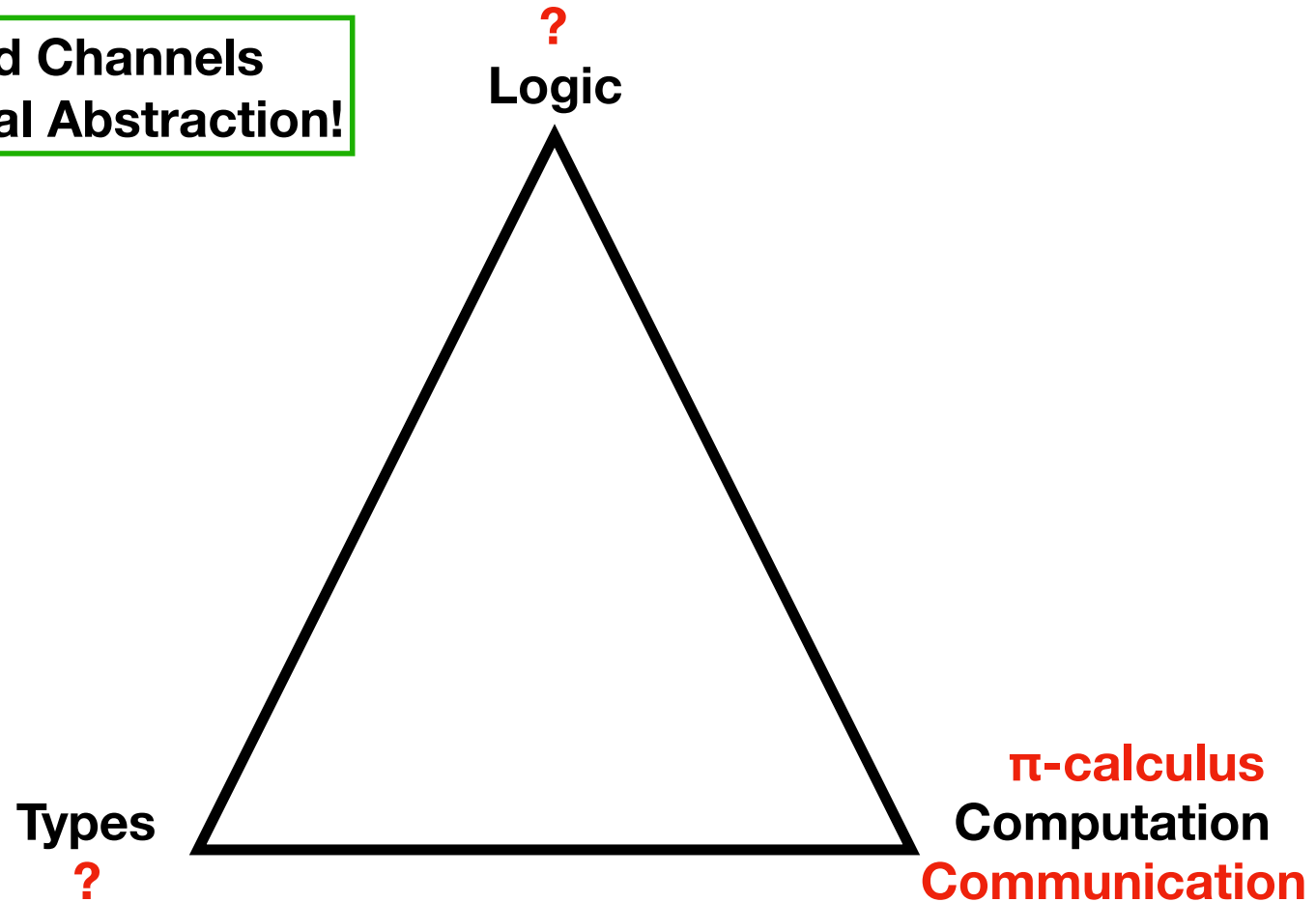
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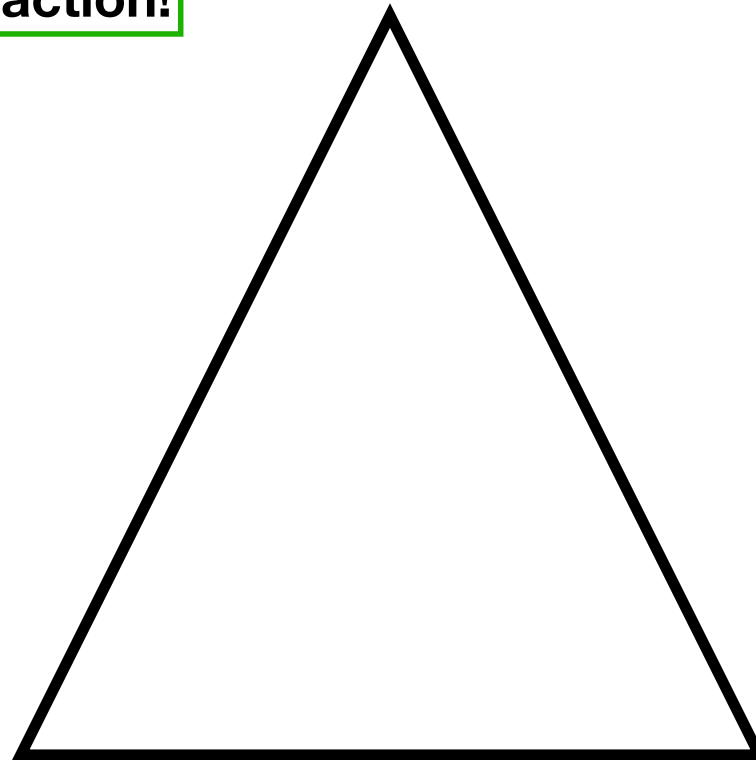
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Session Types!  
[Honda'93]    Types  
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$\pi$ -calculus  
Computation  
Communication



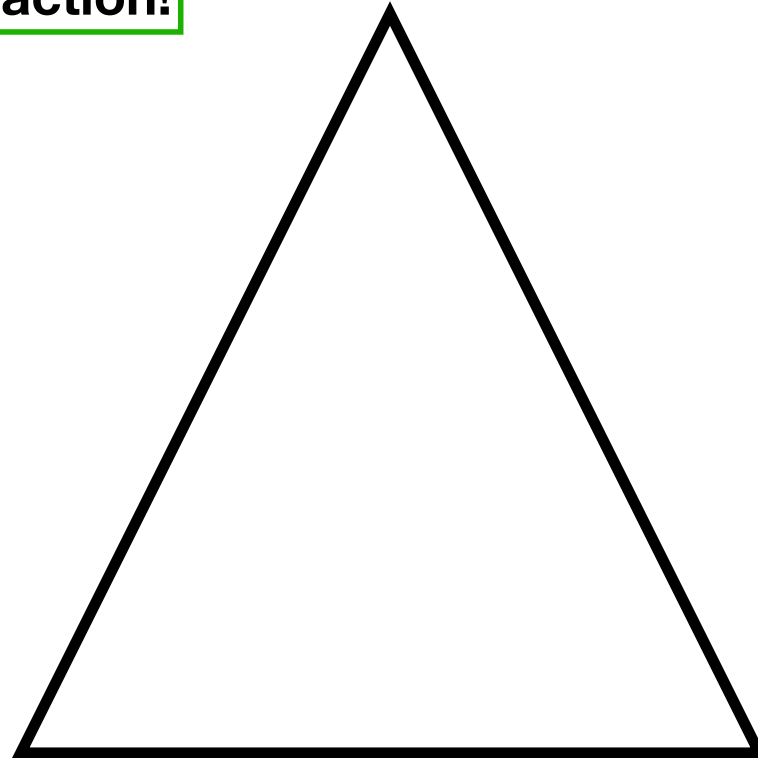
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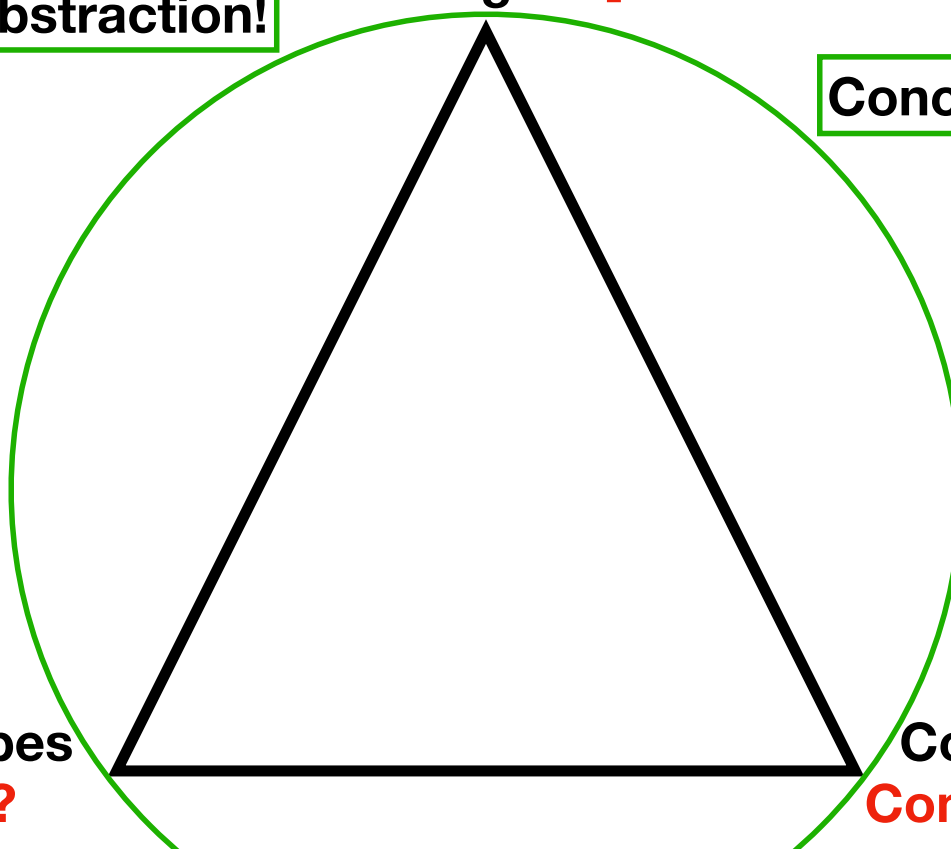
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**There is Hope**

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# There is Hope

**Do not communicate by sharing memory;  
instead, share memory by communicating.  
—Effective Go**

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- From the language point of view: Go
  - Goroutines (threads/processes) as a fundamental abstraction
  - Channels (`chan T`) as a fundamental abstraction
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# Example: A Store (Stack or Queue)

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- Protocol





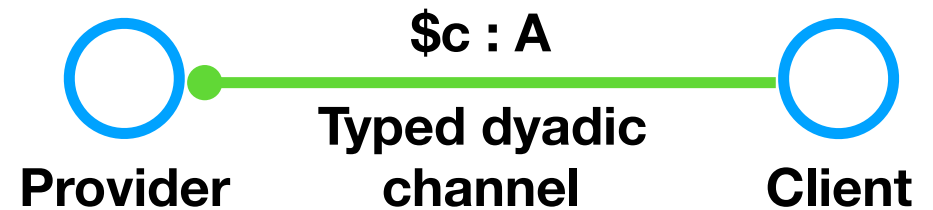
# Example: A Store (Stack or Queue)

- Protocol
  - Client: `ins; x; recurse...`



# Example: A Store (Stack or Queue)

- Protocol
  - Client: `ins; x; recurse...`
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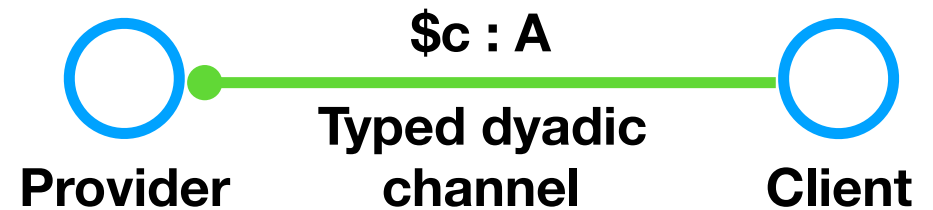
# Example: A Store (Stack or Queue)

- Protocol
  - Client: `ins; x; recurse...`
  - Client: `del`
  - Provider: `none; close.`



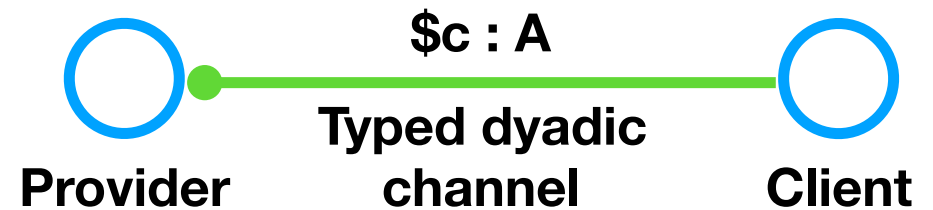
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**Client's choice (external)**

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- Protocol

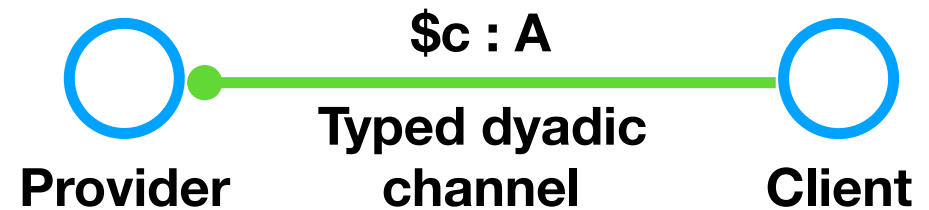
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**Client's choice (external)**

**Provider's choice (internal)**

# A Simple Client, in CC0



# A Simple Client, in C0

```
int main() {
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int main() {  
    int n = 10;
```

# A Simple Client, in CC0

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int main() {  
    int n = 10;  
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int main() {  
    int n = 10;  
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int main() {  
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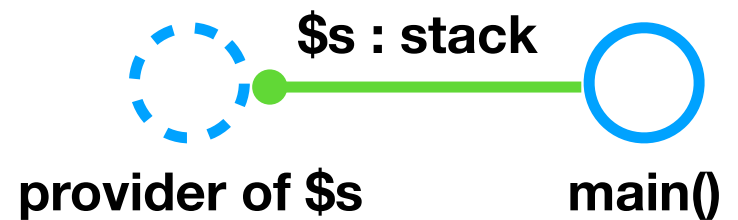
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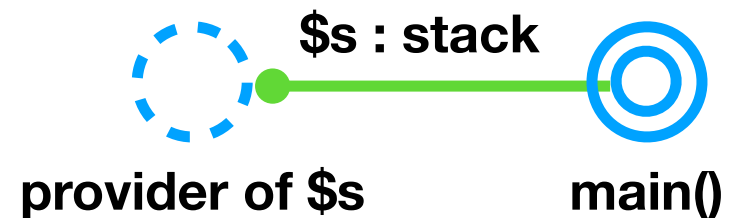
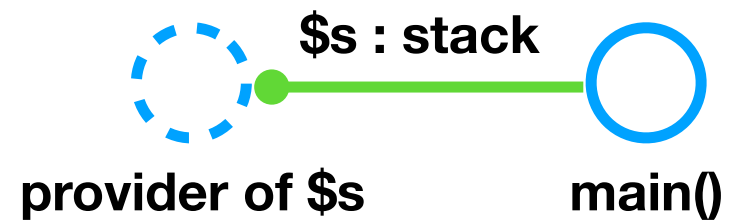
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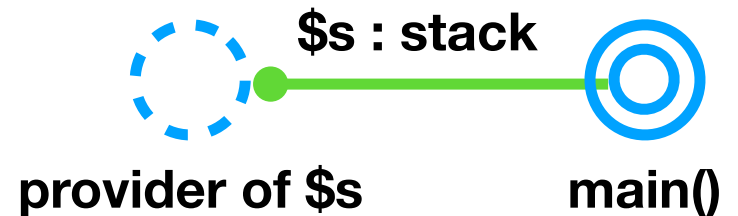
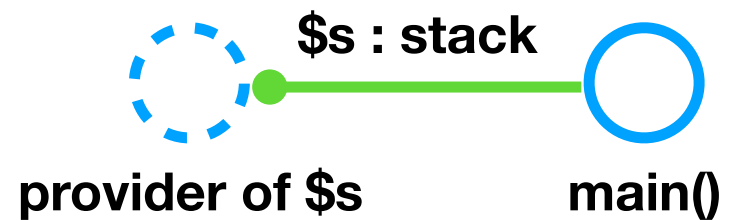
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stack $s elem(int x, stack $t) {  
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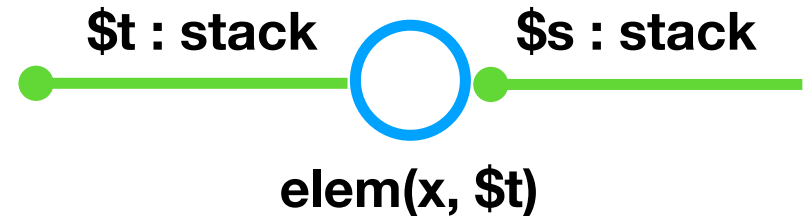
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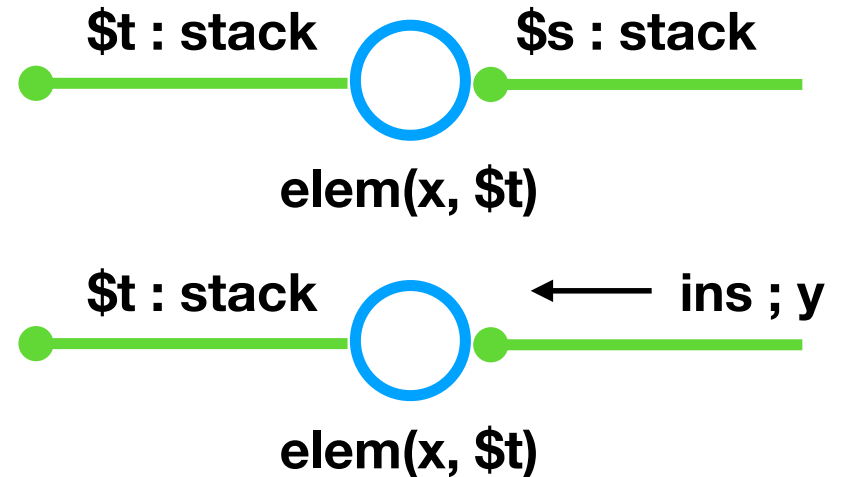
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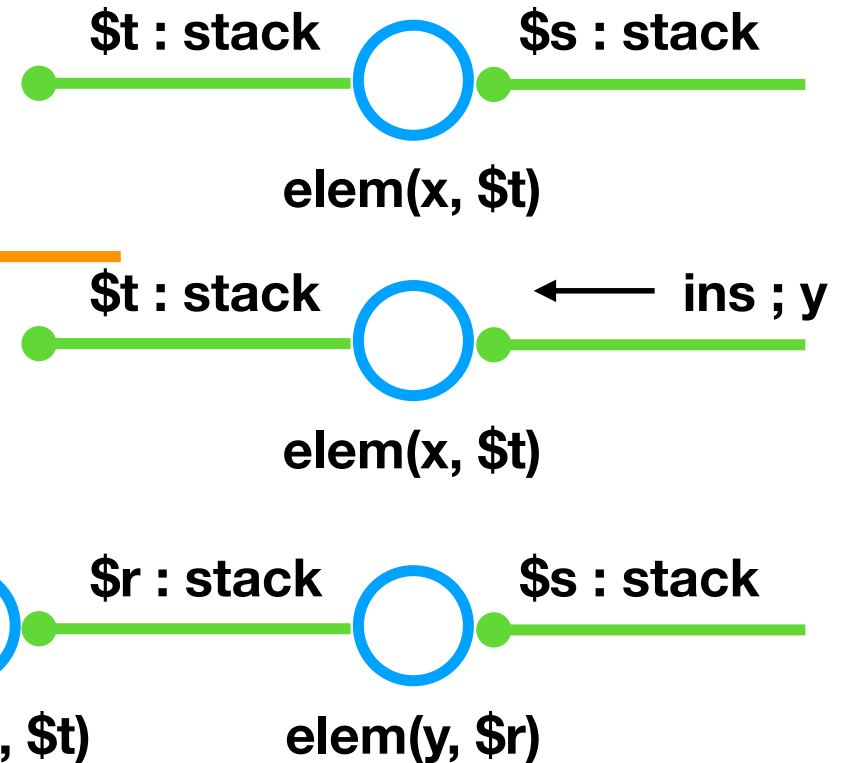
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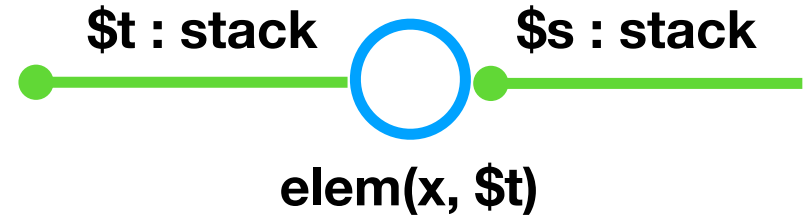






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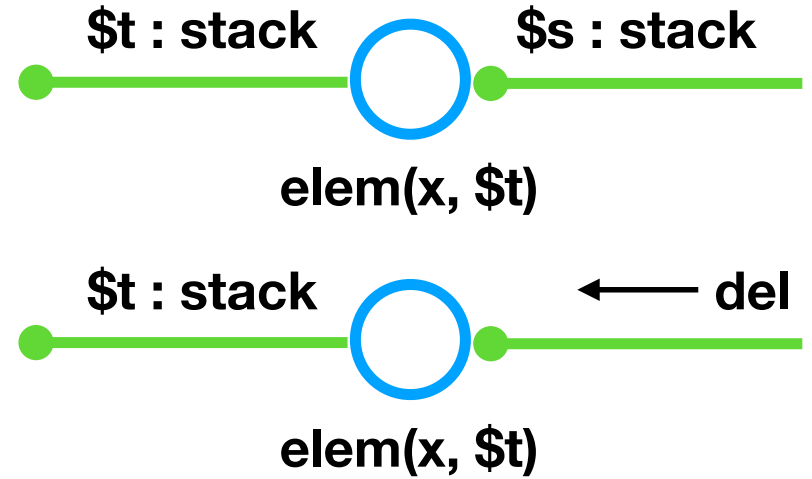
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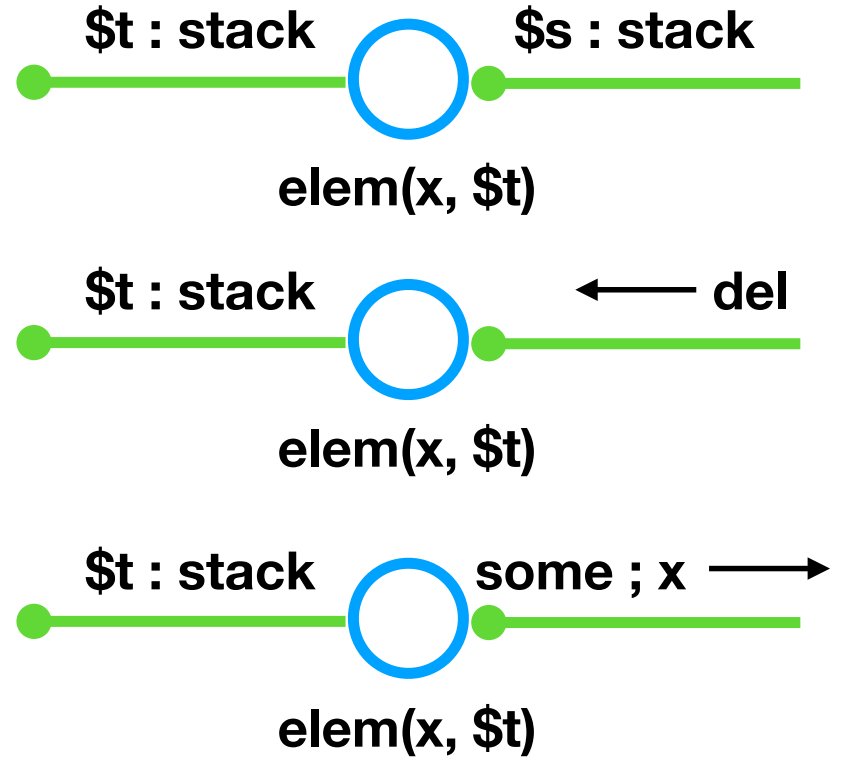
```



```

stack $s elem(int x, stack $t) {
  switch ($s) {
    case ins: {
      int y = recv($s);
      stack $r = elem(x, $t);
      $s = elem(y, $r);
    }
    case del: {
      $s.some;
      send($s, x);
      $s = $t;
    }
  }
}

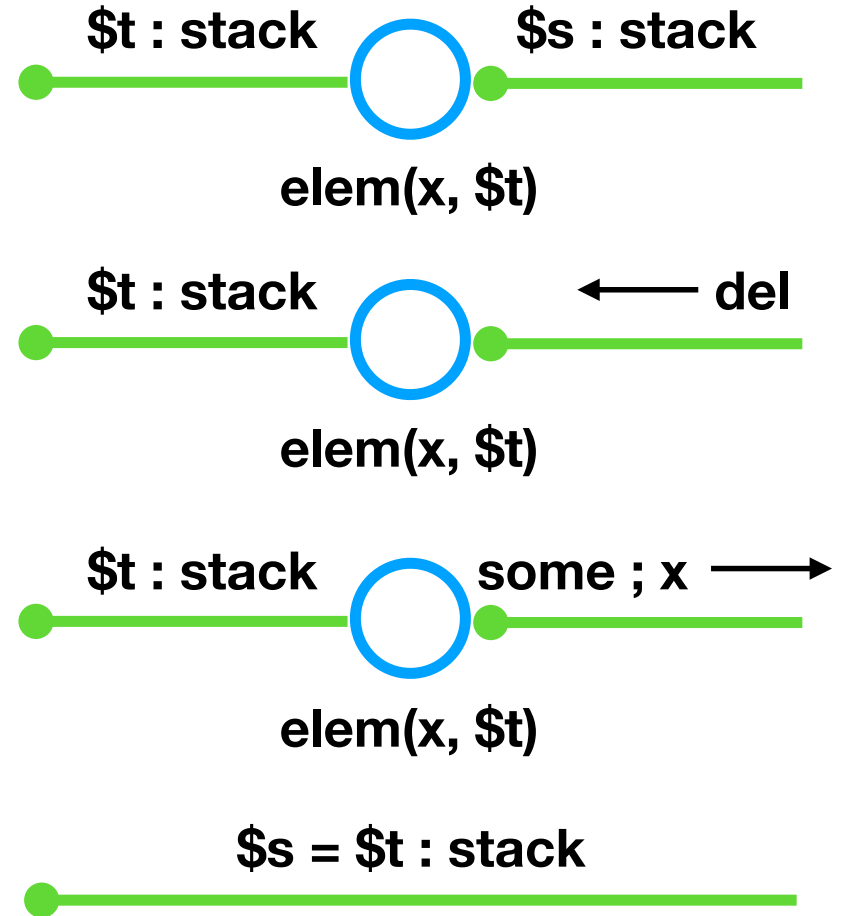
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    }
  }
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```

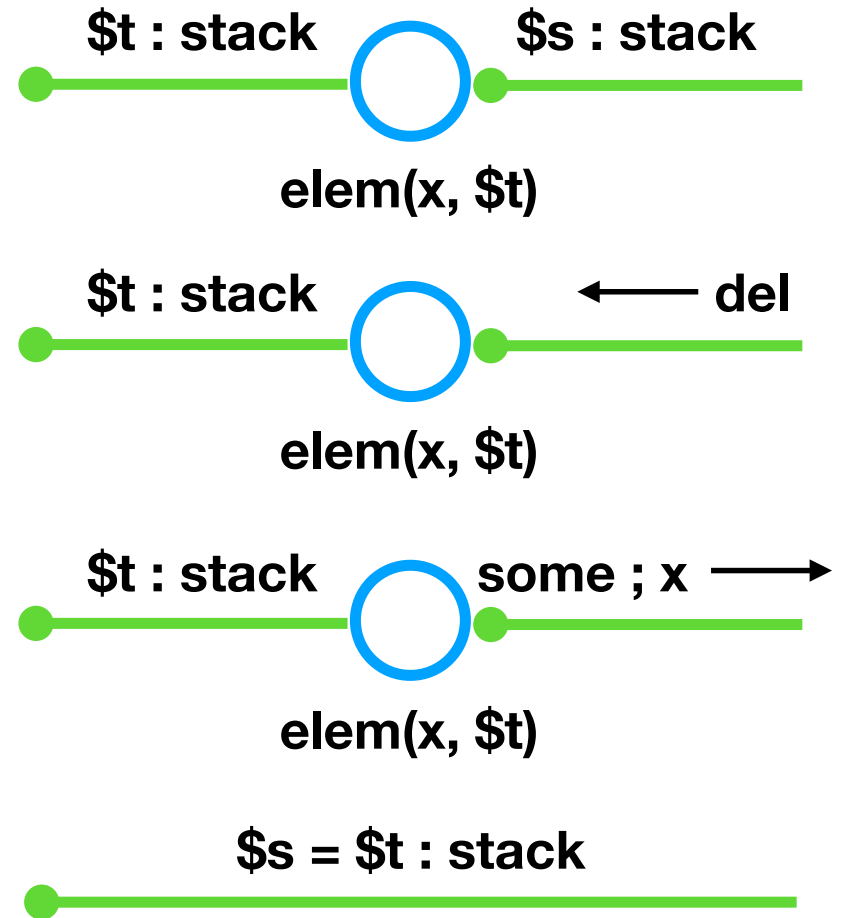


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      send($s, x);
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  }
}

```

**Forwarding (or *channel identification*) is not part of the  $\pi$ -calculus**





```
stack $s empty() {
```



```
stack $s empty() {  
    switch ($s) {
```

```
stack $s empty() {  
  switch ($s) {  
    case ins: {
```

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    }  
  }  
}
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    }  
    case del: {  
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    }  
  }  
}
```



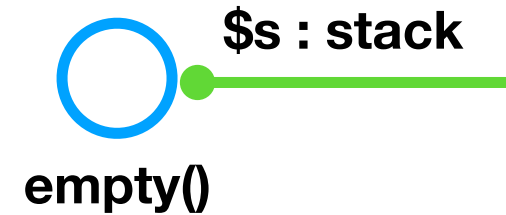
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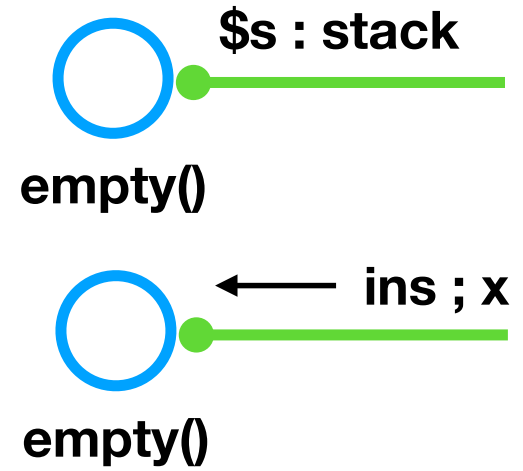
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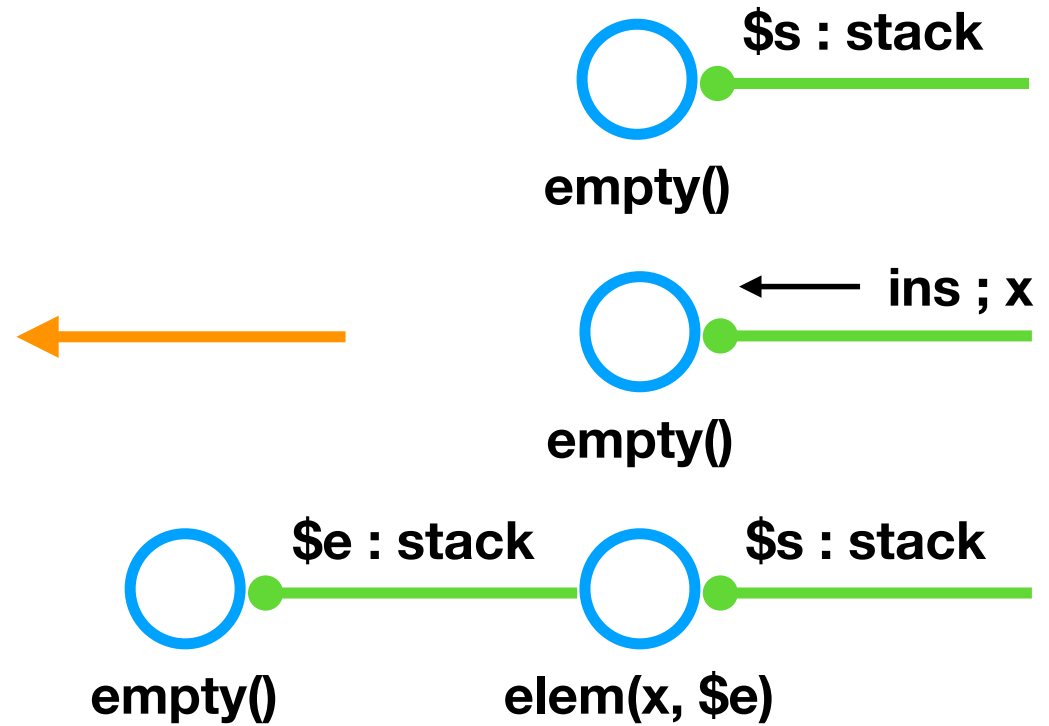
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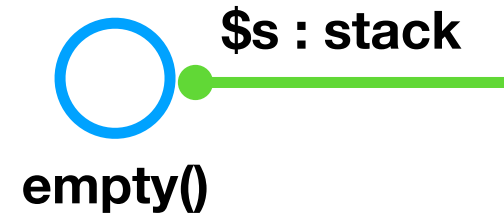




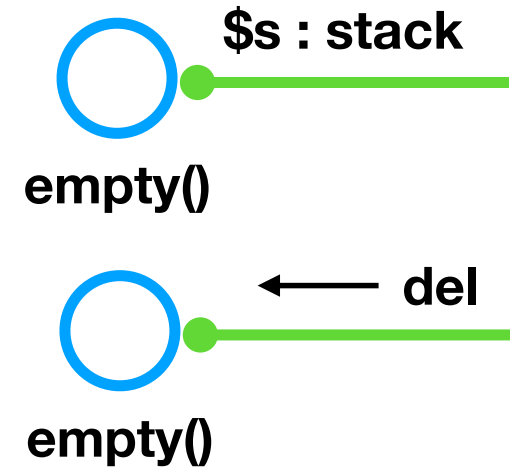


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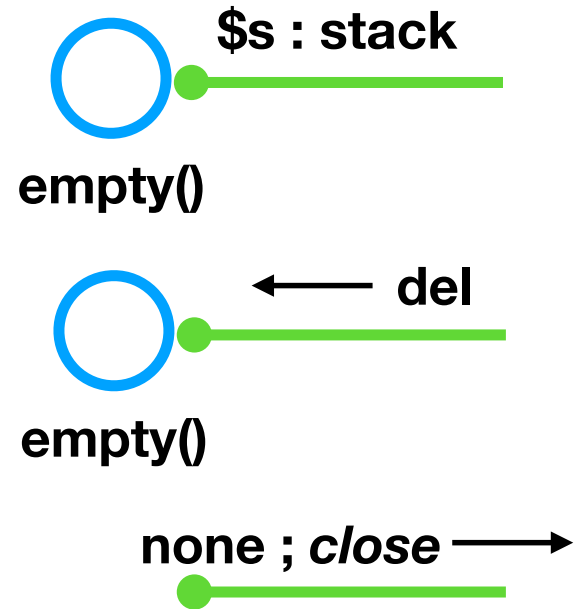
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```



# Summary So Far

- Processes provide one channel and are clients to other channels
- Spawning a process “returns” a fresh channel  $c$ , with two endpoints
  - New process provides  $c$
  - Spawning process is client of  $c$
- Processes can terminate by forwarding
- Communication is bidirectional
  - Processes send and receive labels or integers

# Typing Channels

- Channel types should encode protocol of communication
  - Provider and client must execute complementary actions
- **External choice**: Provider branches on label / Client sends label
- **Internal choice**: Provider sends label / Client branches on label
- **Termination**: Provider terminates / Client waits for termination
- **Basic data**: sending or receiving atomic values

# Session Types, Abstractly

	Type	Provider action	Continuation
$A$	$::=$		
	$\&\{\ell : A_\ell\}_{\ell \in L}$	receive some $k \in L$	$A_k$
	$\oplus\{\ell : A_\ell\}_{\ell \in L}$	send some $k \in L$	$A_k$
	$A \multimap B$	receive channel $c : A$	$B$
	$A \otimes B$	send channel $c : A$	$B$
	$\mathbf{1}$	terminate	<i>none</i>
	$\forall x:\tau. A$	receive $v : \tau$	$[v/x]A$
	$\exists x:\tau. A$	send $v : \tau$	$[v/x]A$

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	$\forall x:\tau. A$	receive $v : \tau$	$[v/x]A$
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$$\begin{aligned}
 stack_A = \&\{ & \text{ins} : A \multimap stack_A, \\
 & \text{del} : \oplus\{ \text{none} : \mathbf{1}, \\
 & \text{some} : A \otimes stack_A \} \}
 \end{aligned}$$



# Types as Propositions

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- These types are exactly the propositions of linear logic, except ‘!’
- An instance of the Curry-Howard correspondence
  - Typing rules correspond to sequent calculus inference rules
  - Programs correspond to process expressions
  - Communication corresponds to cut reduction

# Session Typing Judgments

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**channels used by P**

$$\underbrace{x_1 : A_1, \dots, x_n : A_n}_{\Gamma} \vdash P :: (x : A)$$

**channel provided by P**



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**channels used by P**

$$\underbrace{x_1 : A_1, \dots, x_n : A_n}_{\Gamma} \vdash P :: (x : A) \quad \text{channel provided by P}$$

**Configuration**  $\Omega ::= (P_1 \mid \dots \mid P_n)$

# Session Typing Judgments

**channels used by P**

$$\underbrace{x_1 : A_1, \dots, x_n : A_n}_{\Gamma} \vdash P :: (x : A)$$

**channel provided by P**

**Configuration**  $\Omega ::= (P_1 \mid \dots \mid P_n)$

**Configuration Typing**  $\Gamma \models \Omega :: \Gamma'$

**channels used by  $\Omega$**

**channels provided by  $\Omega$**

# Theoretical Properties

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- Without recursive types and processes
  - Session fidelity (Preservation)
  - Deadlock freedom
  - Termination
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- With recursion
  - Session fidelity (Preservation)
  - Deadlock freedom

**$\Omega$  is *poised* if all processes in  $\Omega$  attempt to communicate along a channel in the external interface**



# Mode of Communication

- Both synchronous and asynchronous communication can be supported
- **Asynchronous**: messages still must appear in order (for session fidelity)
  - Synchronization via polarization of the types
- **Synchronous**: messages can be coded via one-action processes
- Asynchronous seems to be the right default
  - Closer to reasonable implementation
  - Generalizes to channels with multiple endpoints

# Session Types, in CC0

**? = receive**

**! = send**

**; = sequence of interaction**

**<...> session type**

# Session Types, in CC0

```
choice stack_req {  
  <?int ; ?choice stack_req> ins;  
  <!choice stack_response> del;  
};
```

```
choice stack_response {  
  < > none;  
  <!int ; ?choice stack_req> some;  
};
```

```
typedef <?choice stack_req> stack;
```

**? = receive**  
**! = send**  
**; = sequence of interaction**  
**<...> session type**

# Tracing the Type-Checker

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```
stack $s elem(int x, stack $t) {
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  switch ($s) {
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```
stack $s elem(int x, stack $t) {  
  switch ($s) {  
    case ins: {                                % $s : <?int ; stack> -l $t : stack
```





# Tracing the Type-Checker

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stack $s elem(int x, stack $t) {  
  switch ($s) {  
    case ins: {  
      int y = recv($s);  
      stack $r = elem(x, $t);  
    }  
  }  
}
```

% \$s : <?int ; stack> -| \$t : stack  
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% \$s : stack -| \$r : stack

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      % $s : <?int ; stack> -| $t : stack  
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    }
    case del: {                                     % $s : <!choice stack_response> -| ...
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      $s = elem(y, $r);
    }
    case del: {                                     % $s : <!choice stack_response> -| ...
      $s.some;                                       % $s : <!int ; stack> -| $t : stack
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      send($s, x);                                    % $s : stack           -| $t : stack
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    }
    case del: {                                % $s : <!stack_response> -| .
      $s.none;                                % $s : < >           -| .
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% \$s : <!stack\_response> -| .  
% \$s : < > -| .

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      $s = elem(x, $e);
    }
    case del: {
      $s.none;
      close($s);
    }
  }
}
```

% \$s : <?int ; stack> -| .  
% \$s : stack -| .  
% \$s : stack -| \$e : stack  
% \$s : <!stack\_response> -| .  
% \$s : < > -| .

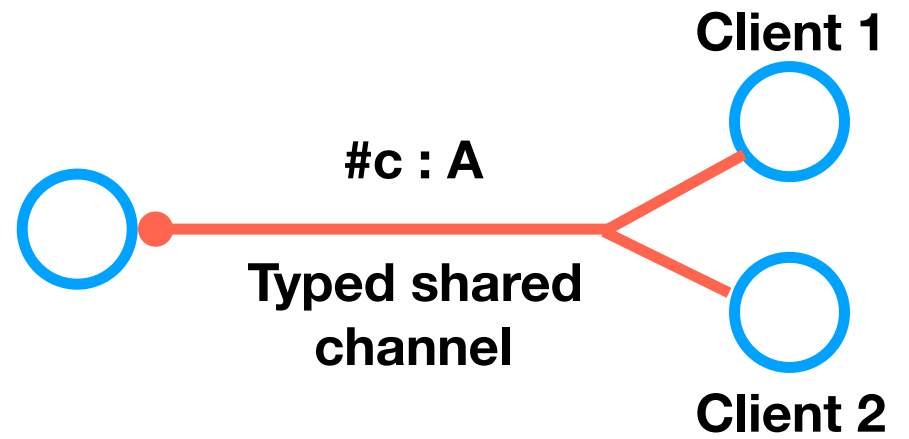
# Tracing the Type-Checker

```
stack $s empty() {
  switch ($s) {
    case ins: {
      % $s : <?int ; stack> -l .
      int x = recv($s); % $s : stack -l .
      stack $e = empty(); % $s : stack -l $e : stack
      $s = elem(x, $e);
    }
    case del: {
      % $s : <!stack_response> -l .
      $s.none; % $s : < > -l .
      close($s);
    }
  }
}
```

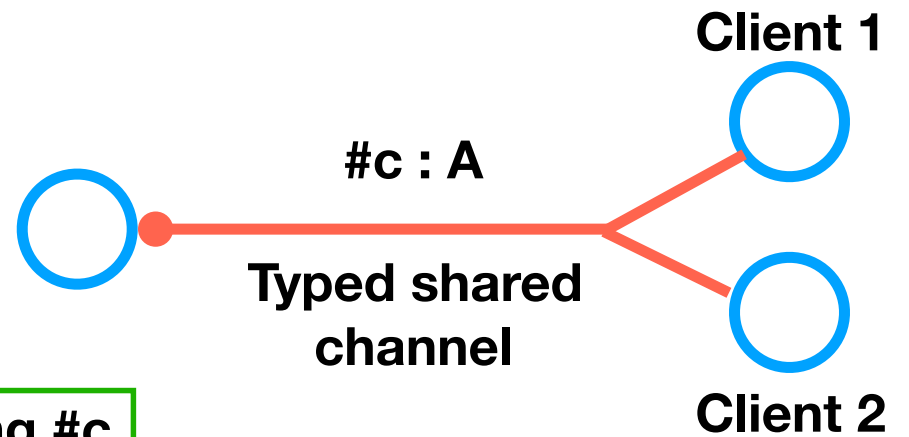


# The Problem with Sharing

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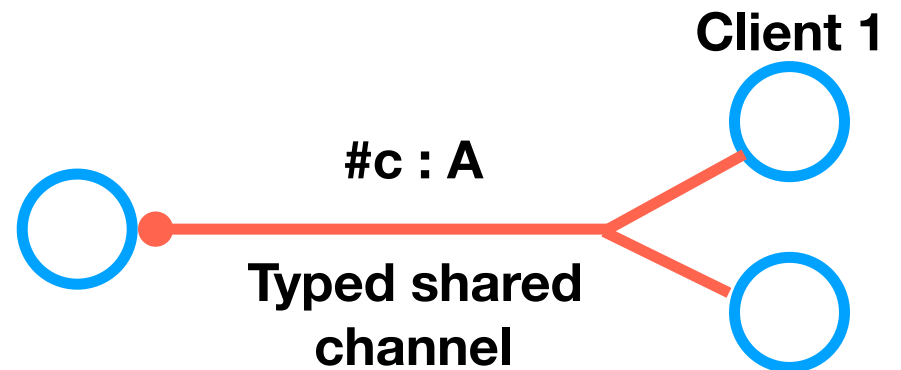


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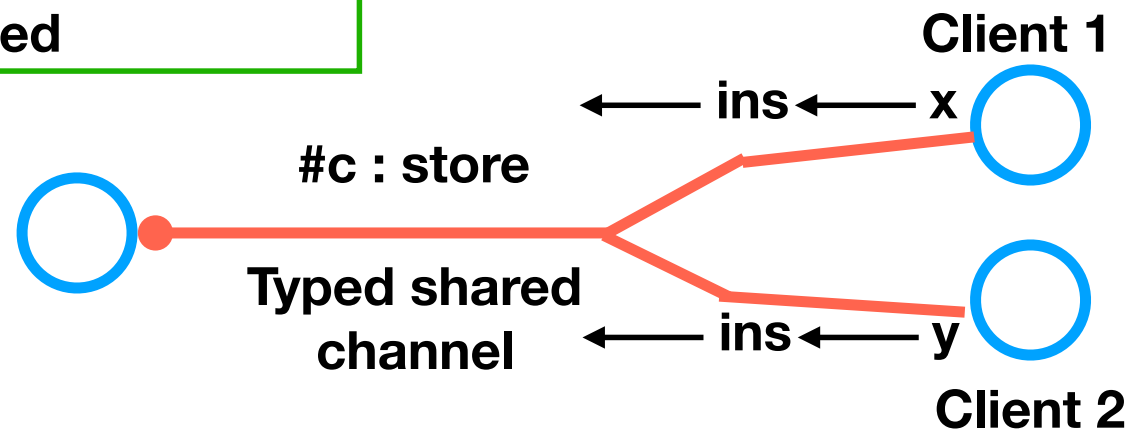


If both clients can freely interact along #c session fidelity will be violated

# The Problem with Sharing



If both clients can freely interact along #c session fidelity will be violated



# Linear and Shared Channels

	Type	Provider action	Continuation
$A$	$::= \&\{l : A_l\}_{l \in L}$	receive some $k \in L$	$A_k$
	$\oplus\{l : A_l\}_{l \in L}$	send some $k \in L$	$A_k$
	$A \multimap B$	receive channel $c : A$	$B$
	$A \otimes B$	send channel $c : A$	$B$
	$\mathbf{1}$	terminate	<i>none</i>
	$\forall x:\tau. A$	receive $v : \tau$	$[v/x]A$
	$\exists x:\tau. A$	send $v : \tau$	$[v/x]A$
	$\downarrow S$	detach from client	$S$
	$S ::= \uparrow A$		accept client

# Linear and Shared Channels

	Type	Provider action	Continuation	
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		$\forall x:\tau. A$	receive $v : \tau$	$[v/x]A$
		$\exists x:\tau. A$	send $v : \tau$	$[v/x]A$
		$\downarrow S$	detach from client	$S$
$S$	$::=$	$\uparrow A$	accept client	$A$
$!A$	$\triangleq$	$\downarrow \uparrow A$		

# A Shared Queue

$$\begin{aligned} \text{queue}_A = \uparrow \& \{ & \text{ins} : A \multimap \downarrow \text{queue}_A, \\ & \text{del} : \oplus \{ \text{none} : \downarrow \text{queue}_A, \\ & \quad \text{some} : A \otimes \downarrow \text{queue}_A \} \} \end{aligned}$$

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The section  $\uparrow \dots \downarrow$  describes a *critical region*



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Certain deadlocks can now arise

Sharing and critical regions are  
manifest in the type!

# Why is Functional Programming So Effective?

Functions are a universal and fundamental abstraction

Intuitionistic Logic

Logic

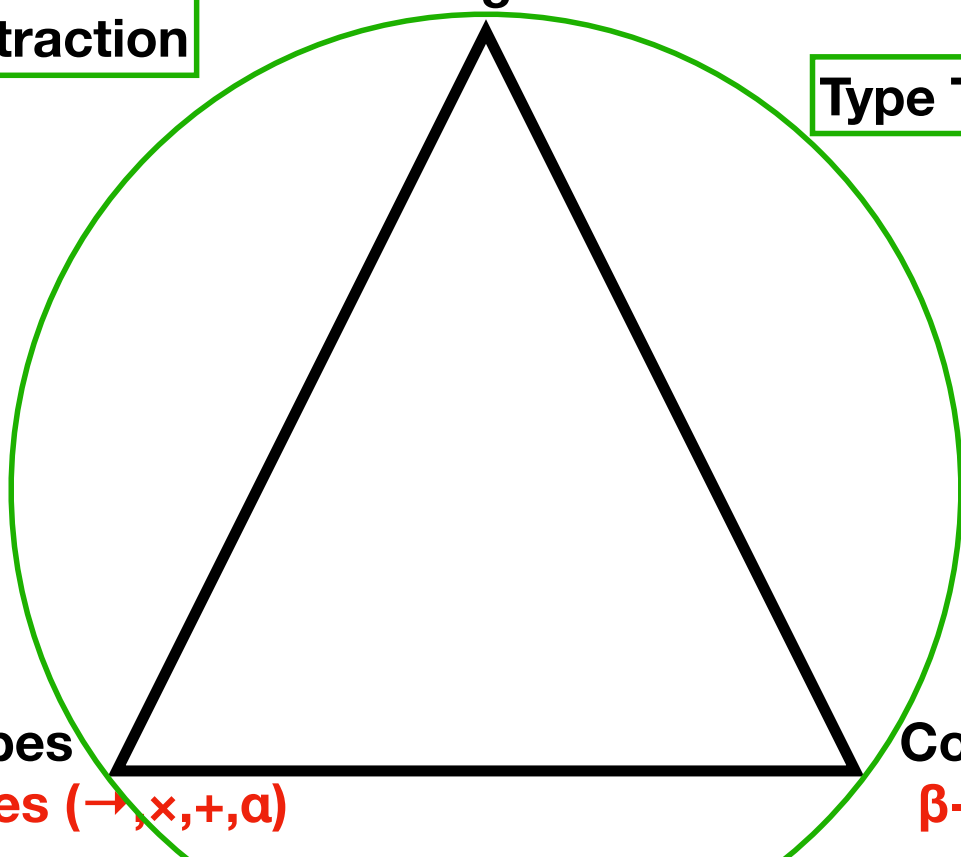
Type Theory

Types

Simple Types ( $\rightarrow, \times, +, \alpha$ )

Computation

$\beta$ -reduction



# What about Concurrency?

Processes and Channels  
are a Fundamental Abstraction!

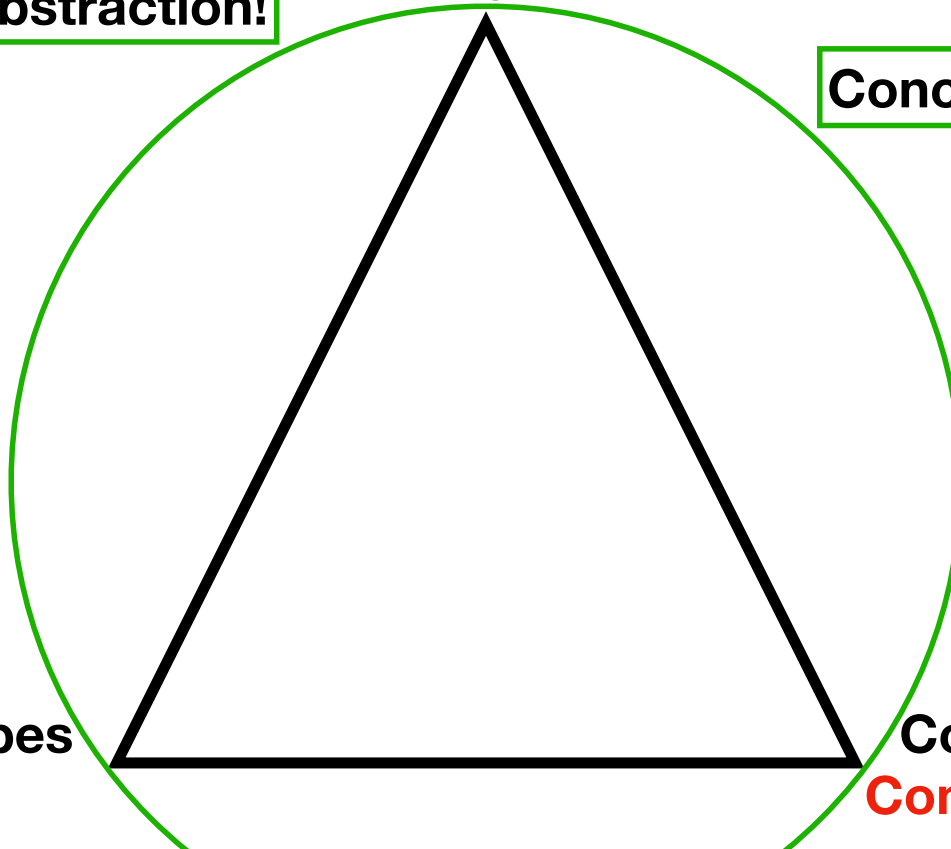
Linear Logic!  
[Caires & Pf'10]

Concurrent Type Theory?

Logic

Session Types!  
[Honda'93] Types

$\pi$ -calculus  
Computation  
Communication



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Processes and Channels  
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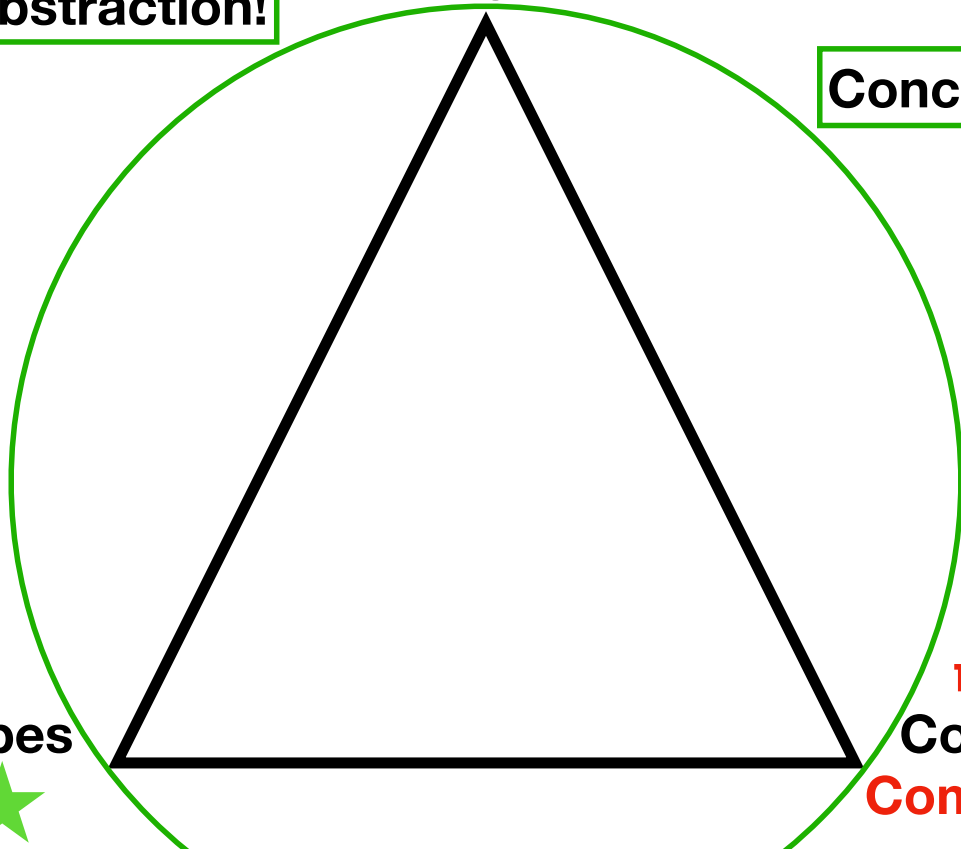
Logic

Session Types!  
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$\pi$ -calculus  
Computation  
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# What about Concurrency?

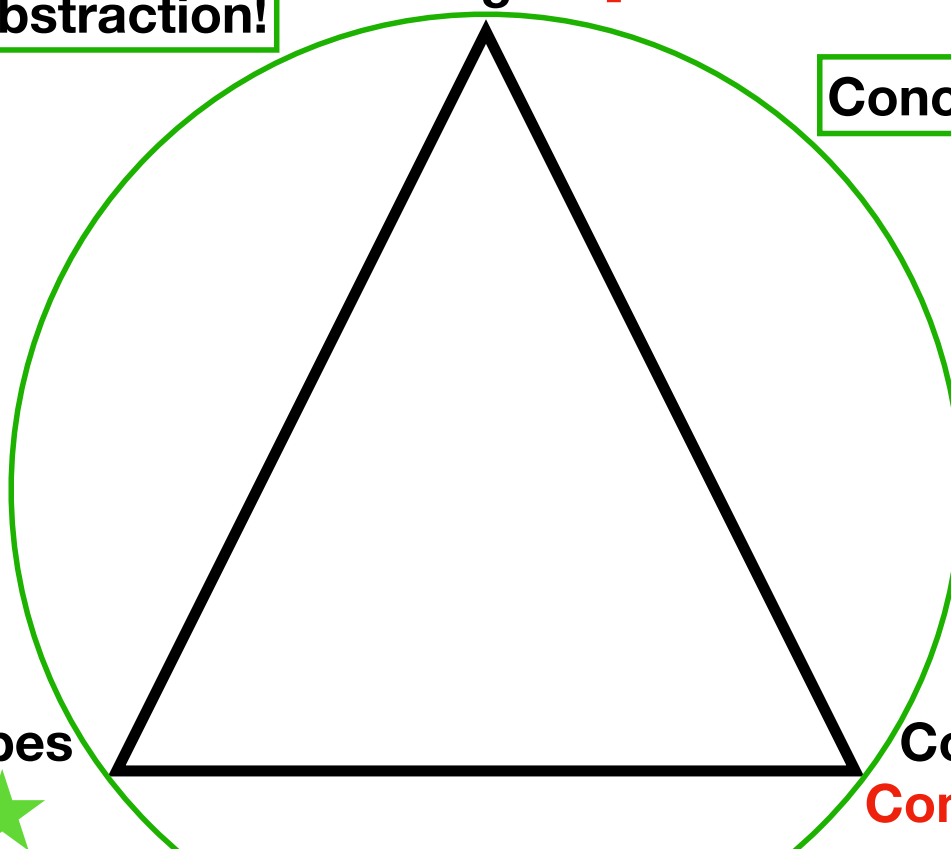
Processes and Channels  
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★  
Logic **Linear Logic!**  
[Caires & Pf'10]

Concurrent Type Theory?

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[Honda'93] ★

**$\pi$ -calculus**  
Computation  
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# What about Concurrency?

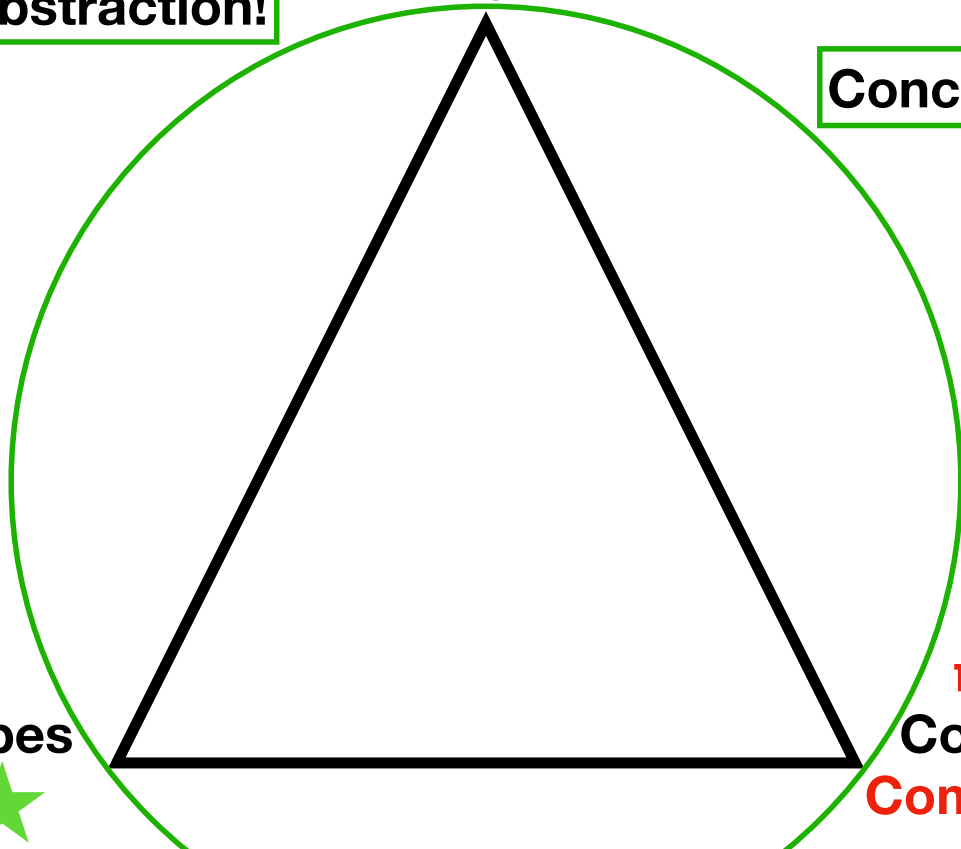
Processes and Channels  
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★  
Logic **Linear Logic!**  
[Caires & Pf'10]

Concurrent Type Theory?

**Session Types!** Types  
[Honda'93] ★

★  
 **$\pi$ -calculus + fwd**  
Computation  
Communication





# Session Types at Present

- Scribble — [www.scribble.org](http://www.scribble.org) — multiparty session types
- ABCD project — [groups.inf.ed.ac.uk/abcd/](http://groups.inf.ed.ac.uk/abcd/) — Simon Gay, Nobuko Yoshida, Philip Wadler
- At CMU — SILL (functional), CC0 (imperative), RSILL (time and work)
- Thanks to my collaborators: Coşku Acay, Stephanie Balzer, Luís Caires, William Chargin, Ankush Das, Henry DeYoung, Anna Gommerstadt, Dennis Griffith, Jan Hoffmann, Limin Jia, Jorge Pérez, Rokhini Prabhu, Klaas Pruiksma, Miguel Silva, Mário Florido, Bernardo Toninho, Max Willsey

# A Paper I Love

- *Types for Dyadic Interaction*, Kohei Honda, CONCUR 1993



**Kohei Honda, 1959–2012**

Types for Dyadic Interaction\*

Kohei Honda

*kohei@mt.cs.keio.ac.jp*

Department of Computer Science, Keio University  
3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223, Japan

## Abstract

We formulate a typed formalism for concurrency where types denote freely composable structure of dyadic interaction in the symmetric scheme. The resulting calculus is a typed reconstruction of name passing process calculi. Systems with both the explicit and implicit typing disciplines, where types form a simple hierarchy of types, are presented, which are proved to be in accordance with each other. A typed variant of bisimilarity is formulated and it is shown that typed  $\beta$ -equality has a clean embedding in the bisimilarity. Name reference structure induced by the simple hierarchy of types is studied, which fully characterises the typable terms in the set of untyped terms. It turns out that the name reference structure results in the deadlock-free property for a subset of terms with a certain regular structure, showing behavioural significance of the simple type discipline.