Higher-Order and Modal Logic as a Framework for Explanation-Based Generalization

Scott Dietzen and Frank Pfenning

School of Computer Science
Carnegie Mellon University
Pittsburgh, Pennsylvania 15213-3890

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Abstract

Logic programming provides a uniform framework in which all aspects of explanation-based generalization and learning may be defined and carried out, but first-order Horn logic is not well suited to application domains such as theorem proving or program synthesis where functions and predicates are the objects of computation. We explore the use of a higher-order representation language and extend EBG to a higher-order logic programming language. Variables may now range over functions and predicates, which leads to an expansion of the space of possible generalizations. We address this problem by extending the logic with the modal $\Box$ operator (indicating necessary truth) which leads to the language $\lambda^\Box\text{Prolog}$. We develop a meta-interpreter realizing EBG for $\lambda^\Box\text{Prolog}$ and give some examples in an expanded version of this extended abstract which is available as a technical report [2].

1 Introduction

To date, work on the explanation-based approach to learning relies primarily on *explanation-based generalization* (EBG) as its central mechanism [8, 1]. Recently, the logic programming paradigm has been used as a foundation for supporting EBG [7, 11, 5]. One argument put forward in favor of the logic programming framework is that it admits a uniform representation for all aspects of EBG: domain theory, training instance, goal, goal concept, operationality definition, *etc.* This helps in explicating the underlying principles in a uniform way and clarifies semantic issues.

Generalization and learning performance are intimately tied to the underlying language for representation, or *representation domain*. If knowledge is encoded in an inappropriate representation domain, then it is less likely that the desired generalizations can be expressed in a natural and concise manner, and also less likely that they can be found.

In this paper we explore two ways of enriching the representation domain of Horn logic, while staying within the logic programming framework: integrated support for higher-order objects including variables ranging over such objects (functions and predicates) and support for modal concepts. Both of these have a significant impact on EBG. We concentrate on how a rich representation language supports EBG, and largely ignore questions about how these generalizations may be assimilated and applied automatically. For a discussion of these issues, see Frieditis [11], and Donat & Wallen [4]. We also do not deal with the important concept of operationality; see Hirsh [5] for a treatment in the context of logic programming.

We begin by motivating the step to a higher-order language as the representation domain. We then address the intractability of the space of possible higher-order generalizations by differentiating domain theory and training instance in a ‘logical’ way. As λProlog is not initially suited to supporting this distinction, we extend the language with the modal operator $\Box$ forming $\lambda^\Box\text{Prolog}$. We then develop an implementation of EBG within $\lambda^\Box\text{Prolog}$ through a generalizing meta-interpreter written in λProlog .

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2 Higher-order EBG and Representation Domains

A domain is *higher-order* if it contains higher-order values such as functions or predicates. A language or application over such a domain is itself said to be higher-order. For example, a higher-order programming language allows functions to be bound to variables, passed as parameters, and returned from function calls. Similarly, a higher-order logic provides for quantification over functions and predicates. *Higher-order EBG* is explanation-based generalization in which the candidates for variable replacement include higher-order objects.

A *representation domain* is a language for expressing application domain values. When higher-order domains are expressed in first-order languages, reasoning and programming over these *ad hoc* encodings is difficult: We often need “new variables”, need to check conditions such as “where ... does not occur in ...”, or must implement substitution in a way that “renames bound variables if necessary.” In this paper we explore the use of *higher-order* representation domains that provide for natural expression and manipulation of higher-order objects. Consider the following higher-order rule for reducing an integration task by extracting a constant factor a :

$$\forall a \forall f \forall g. \int f = g \Rightarrow \int \lambda x. a * f(x) = \lambda x. a * g(x)$$

(λ is used to explicitly express function abstraction.) Within a higher-order representation domain, that is, one that supports name binding (via λ), the restriction that x not be free in the expression a is captured without complicating side conditions.

Many other domains naturally involve binding constructs and are thus best expressed in a higher-order representation domain: logics (when viewed as domains), programming languages, and natural language [10, 9]. Similar representational needs arise when one wants to reason “at the meta-level”, that is, about control strategies for logic programming, theorem proving, or the EBG algorithm itself: One would like facts (propositions) or properties (predicates) to be objects themselves. Prolog and other first-order languages allow this to some extent, but in a way that is only operationally, but not logically motivated. This complicates reasoning about these concepts and, in practice, often prohibits applications such as EBG.

Nadathur & Miller describe λ Prolog, a logic programming language supporting higher-order functions and predicates [9]. It is based on higher-order hereditary Harrop formulas which generalize Horn clauses while still preserving the basic properties of the logic programming paradigm. λ Prolog serves simultaneously as our representation domain and as the implementation language for EBG.

3 EBG and Modal Logic

When describing domain theory and training instance, we distinguish the former with the $\Box$ operator. $\Box$ is borrowed from modal logic (see, for example, Hughes & Cresswell [6]). It precedes *necessarily* true sentences, or equivalently, those true in *all worlds*; non-prefixed sentences are only *contingently* true, or true in the *current world*. Previous EBG implementations have differentiated between domain theory and training instance either by explicit separation [8] or by distinguishing rules (implications) from facts (atomic clauses) [7, 5, 11]. A modal logic representation subsumes the latter and can also be given a clear semantics

which is independent of a particular generalization algorithm or search procedure. We illustrate the use of $\Box$ with a first-order example from [1]:

$$\begin{aligned} \Box \forall a \forall b \forall c. \text{hate}(a, b) \wedge \text{possess}(a, c) \wedge \text{weapon}(c) &\Rightarrow \text{kill}(a, b) \\ \Box \forall w. \text{depressed}(w) &\Rightarrow \text{hate}(w, w) \\ \Box \forall u \forall v. \text{buy}(u, v) &\Rightarrow \text{possess}(u, v) \\ \Box \forall z. \text{gun}(z) &\Rightarrow \text{weapon}(z) \\ \text{depressed}(\text{john}) & \\ \text{buy}(\text{john}, \text{gun1}) & \\ \text{gun}(\text{gun1}) & \end{aligned}$$

The generalization resulting from the query $\text{kill}(\text{john}, \text{john})$ may be expressed as

$$\Box \forall a \forall b. \text{depressed}(a) \wedge \text{buy}(a, b) \wedge \text{gun}(b) \Rightarrow \text{kill}(a, a)$$

Clauses of the training example (*i.e.*, those without a $\Box$ prefix) can safely be removed without invalidating the generalization. Revising or replacing the training instance can be explained semantically as switching worlds. Since $\Box$ is not restricted to outermost occurrences, one has the potential to intermix domain and training knowledge.

The higher-order and modal aspects of our representation domain work well together: Through the use of $\Box$ to distinguish domain theory and training example, we elegantly restrict the space of generalizations for higher-order EBG. This generalization space is much larger than that for first-order EBG, since functions and predicates are also candidates for variable replacement. Our higher-order EBG algorithm is sufficient to determine goal concepts for published examples (such as the above) and allows direct application to higher-order domains such as theorem proving and program derivation.

As we have made $\Box$ part of our logic, it must become part of the underlying logic programming language. The expanded language, $\lambda^\Box\text{Prolog}$, is implemented with an interpreter written in λProlog . The concise and elegant extension of this interpreter to perform EBG along the lines of [7, pages 387–388] is presented in full in [2].

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