15-453 Formal Languages, Automata, and Computation

Review Problems 2

Friday, April 14

Problem 1

Explore the properties of the $\leq_P$ relation. In particular, check whether it is reflexive, symetric and antisymetric. In cases where the answers are negative, show counter-examples.

Problem 2

A language L is complete for a class C with respect to polynomial time reduction if $L \in C$ and $\forall L' \in C \quad L' \leq_P L$. Show that:

a) Σ^* and $\emptyset$ are the only languages in P that are not complete for P.

b) L is complete for NP iff L^c is complete for co-NP.

Problem 3

Exhibit a polynomial time reduction from SUBSET-SUM (defined on page 246 of the textbook) to INTLIN $= \{E \mid E$ is a solvable system of integral linear inequalities$\}$.

Problem 4

Let ϕ be a boolean formula constructed from the boolean input variables $x_1, x_2, \ldots, x_k$, negations $(\neg)$, AND's $(\wedge)$, OR's $(\vee)$, and parentheses. The formula ϕ is a *tautology* if it evaluates to 1 for every assignment of 1 and 0 to the input variables. Define TAUTOLOGY as the language of boolean formulas that are tautologies. Show that TAUTOLOGY $\in$ co-NP.

Problem 5

As defined on page 261 if the textbook, *vertex cover* of an undirected graph G is a subset of nodes where every edge of G touches one of those nodes. Let

$$\text{VERTEX-COVER} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover}\}$$

Show that VERTEX-COVER $\in$ NP and exhibit a polynomial time reduction from CLIQUE to VERTEX-COVER. (Note: the book shows the NP-completeness of VERTEX-COVER by reducing from 3SAT).

Problem 6

Given a language class C, we define a class co-C $= \{L^c \mid L \in \mathrm{C}\}$. Show that

(a) if $C_1 \subseteq C_2$ then co-$C_1 \subseteq$ co-C_2

(b) P = co-P

(c) P $\subseteq$ co-NP