15-453 Formal Languages, Automata, and Computation

Midterm II

March 24, 2000

Name: ______________________________

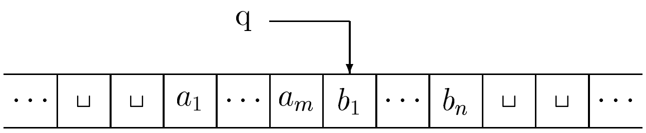
Andrew ID: ______________________________

- This is a closed book exam; only 1 sheet of notes is permitted.
- Write your answer legibly in the space provided.
- You have 50 minutes for this exam.
- This exam consists of 3 questions worth a total of 100 points, plus an extra credit question worth 40 points.
- Extra credit is recorded separately, so make sure to solve Problems 1–3 completely before moving on to extra credit.

Problem 1	Problem 2	Problem 3	Total	Extra Credit
25	35	40	100	40

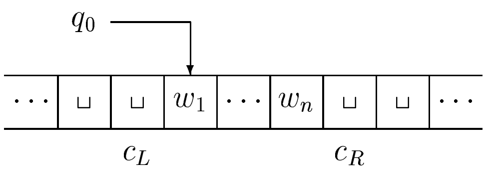
Problem 1 (25 pts)

A Turing machine with a 2-way infinite tape (TM2) is like a regular Turing machine, except that we think of a configuration $a_1 \cdots a_m q b_1 \cdots b_n$ as representing the tape



The initial configuration is still $q_0 w$ for the initial state q_0 and input word w, but the transition relation at the left end of the current configuration works symmetrically to the one at the right end, filling in blanks ␣ as needed.

We say a machine *uses a cell* if the read/write head points to the cell at least once during the computation of a machine. We use c_L to denote the cell to the left of the input (initially containing ␣) and c_R the cell to the right of the input (also initially containing ␣)



For each of the following questions about a TM2 M and input w, check **all** boxes that apply. You do not need to justify your answers.

		decid-able	recogniz-able	co-recogniz-able
a.	M uses c_L			
b.	M uses c_R			
c.	M uses c_L or c_R			
d.	M uses only a finite number of cells			
e.	M uses an infinite number of cells			

Problem 2 (35 pts)

Prove carefully that the acceptance problem

$$A_{TM2} = \{\langle M, w\rangle \mid M \text{ is a TM2 accepting } w\}$$

is undecidable. You may use a mapping reduction or a proof by contradiction. In either case you should clearly state your proof method and explain your construction in detail.

Problem 3 (40 pts)

a. Is the acceptance problem for context-free grammars

$$A_{CFG} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates } w\}$$

decidable? You only need to answer yes or no (this was covered in class).

b. Is the problem

$$ALL_{CFG} = \{\langle G\rangle \mid G \text{ is a CFG with } \mathcal{L}(G) = \Sigma^*\}$$

decidable? You only need to answer yes or no (this was covered in class).

c. Prove that

$$EQ_{CFG} = \{\langle G_1, G_2\rangle \mid G_1 \text{ and } G_2 \text{ are CFGs with } \mathcal{L}(G_1) = \mathcal{L}(G_2)\}$$

is undecidable. You may use the answers to (a) and (b).

d. Prove that EQ_{CFG} is co-Turing-recognizable. You may use the answers to (a) and (b).

Problem 4 (40 pts extra credit)

A *context-sensitive grammar* (CSG) $C = (V, \Sigma, R, S)$ is like a context-free grammar, except that the rules may have the form

$$xAy \longrightarrow w$$

for $x, y \in \Sigma_\epsilon$, $A \in V$, and $w \in (V \cup \Sigma)^*$. We permit a derivation step $uxAyv \Longrightarrow uwv$ if $xAy \longrightarrow w$ is in R. Note that x and y may either be single letters in Σ or empty, so a context-free grammar is a special case of a context-sensitive grammar. As before

$$\mathcal{L}(C) = \{w \mid S \Longrightarrow \cdots \Longrightarrow w \text{ for } w \in \Sigma^*\}$$

Prove that the emptiness problem for context-sensitive grammars

$$E_{CSG} = \{\langle C\rangle \mid C \text{ is a CSG and } \mathcal{L}(C) = \emptyset\}$$

is undecidable by a reduction from the acceptance problem for Turing machines with two-way infinite tapes, A_{TM2}.
Hint: Represent states of the Turing machine by non-terminals and configurations by strings.

Worksheet

Worksheet

Worksheet

Worksheet