15-453 Formal Languages, Automata, and Computation

Midterm Examination

February 18, 2000

Name: _____

Andrew ID: _____

- This is a closed book exam; only 1 sheet of notes is permitted.
- Write your answer legibly in the space provided.
- You have 50 minutes for this exam.
- This exam consists of 2 questions worth 50 points each, plus a third question for extra credit.
- The extra credit is recorded separately, so make sure you have answered questions 1 and 2 correctly before moving on to question 3.

Problem 1	Problem 2	Total	Extra Credit
50	50	100	30

Problem 1 (50 pts)

Consider the regular expression $R = (a \cup \epsilon) \circ (ab \cup bc \cup ac)^*$ over the alphabet $\Sigma = \{a, b, c\}$.

a. (25 pts) Construct an NFA N with 4 states recognizing the language generated by R. You only need to show the state diagram for N, not a mathematical definition. If you need more than 4 states, you will receive partial credit.

b. (25 pts) Using the procedure presented in class, convert your NFA to a DFA. Again, you only need to show the state diagram. You will receive partial credit for a correct DFA, even it has not been constructed systematically from the NFA in part (a).

Problem 2 (50 pts)

We define the operation $\overline{a}$ for $a \in \{0,1,2\}$ by $\overline{0} = 2$, $\overline{1} = 1$, and $\overline{2} = 0$. We extend this to words by defining $\overline{a_1 \dots a_n} = \overline{a_1} \dots \overline{a_n}$. Now consider the language

$$L = \{w\overline{w}^R \mid w \in \{0,1,2\}^*\}$$

where $\overline{w}^R$ is the reverse of the word $\overline{w}$.

a. (15 pts) Show a context-free grammar G which generates L.

b. (15 pts) Show the state diagram of a PDA P recognizing L.

c. (20 pts) Using the pumping lemma for regular languages, prove carefully that L is not regular.

Problem 3 (30 pts extra credit)

Is the language $\{0^n 1^{2n} \mid n \geq 0\}$ context-free? Prove your answer!

Worksheet

Worksheet

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