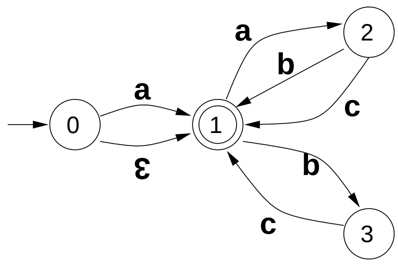
15-453 Formal Languages, Automata, and Computation

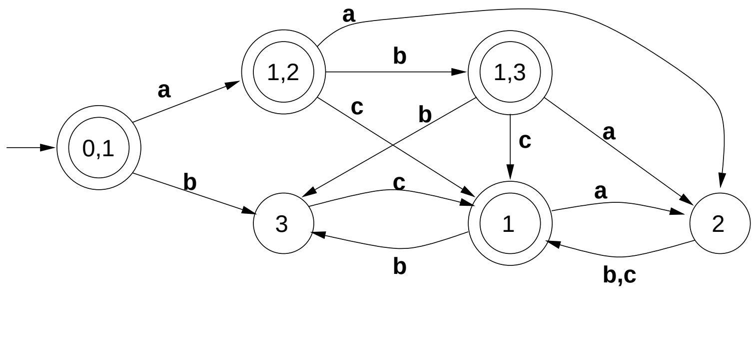
Solutions of the First Midterm

Problem 1

a)



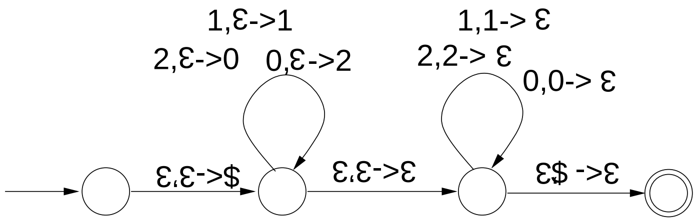
b) The figure shows the result of the subset construction; only the reachable states are shown. All missing arcs lead to a non-final state used to reject.



Problem 2

a) $S \to 1S1 \mid 0S2 \mid 2S0 \mid \epsilon$.

b)



c) Assume L is regular. Let p be the pumping length. Take the word $w = 0^p 1^p \in L$. By the pumping lemma w can be written as $w = xyz$, where $|y| > 0$ and $|xy| \leq p$. This means $y = 0^k$ for some $k > 0$. The pumping lemma also says that $xz = 0^{p-k}1^p \in L$, but this is a contradiction. So L cannot be regular.

Problem 3

The language is context-free. Here is a grammar which generates it:
$S \to 0S11 \mid \epsilon$.

Proof:

$L \subseteq \mathcal{L}(S)$ By induction over the length of $w \in L$. The base case is trivial, as $S \Rightarrow \epsilon$. By induction, if $w \in L$, and $|w| > 0$, then $w = 0w'11$ where $w' \in L$. But then by the induction hypothesis $S \overset{*}{\Rightarrow} w'$, so $S \overset{*}{\Rightarrow} w$.

$\mathcal{L}(S) \subseteq L$ We can prove by induction on the length of the derivation of a $S \overset{*}{\Rightarrow} w$ that $w \in L$. The base case, a derivation of one step, implies $w = \epsilon \in L$. If $S \Rightarrow 0S11 \overset{*}{\Rightarrow} 0w'11 = w$, then by the induction hypothesis $w' \in L$, but then $w = 0w'11 \in L$ too.