15-453 Formal Languages, Automata, and Computation

Homework 10, due April 28

Problem 1 (35 points)

A formula ϕ is in disjunctive normal form (DNF) if it is expressed as an OR of clauses, each of which is the AND of one or more literals (i.e. this is like CNF-form, but swap $\wedge$ and $\vee$). Let DNF-SAT be the set of satisfiable formulas in DNF-form.

(a) Prove that DNF-SAT $\in$ P. [Hint: for a conjunction to be true, all its literals have to be true.]

(b) Prove that DNF-SAT $\subseteq$ SAT.

(c) The problem SAT is NP-complete. Does then (b) imply that DNF-SAT is also NP-complete? Why or why not?

(d) It can be shown that any boolean formula can be written in DNF-form[1]. Let f_{DNF} be a function that maps a given formula into its DNF-form; $f(\phi) = \phi_{DNF}$. if ϕ is satisfiable, $f_{DNF}(\phi)$ is too, so f_{DNF} is a reduction from SAT to DNF-SAT. Knowing that DNF-SAT $\in$ P, does this imply that SAT $\in$ P (and thus P = NP)? Why or why not?

Problem 2 (35 points)

Let us assume that we can solve SAT in polynomial time (i.e. P=NP). Show how we can solve the following problems in polynomial time:

[1] I.e. for each formula $\phi(x_1, x_2, \ldots, x_n)$ there exists a formula $\phi_{DNF}(x_1, x_2, \ldots, x_n)$ in DNF-form such that $\phi(x_1, x_2, \ldots x_n) = \phi_{DNF}(x_1, x_2, \ldots, x_n)$ for all possible values of $x_1, \ldots, x_n$.

1. [Problem 7.29 in the book:] Given a boolean formula ϕ, print a satisfying assignment, or print "none" if none exists. **Hint:** try to find a the right values for each variable one by one.

2. Find the biggest clique in a given graph. **Hint:** use the fact that k-CLIQUE is NP-complete and search for the biggest k.

Problem 3 (30 points)

For the following two problems, prove that if each can be solved in polynomial time, then P=NP. **Hint:** Assume there is a poly-time algorithm for the problem. Build then a poly-time algorithm for some NP-complete problem. Notice that these two aren't *decision* problems, so you cannot simply use a reduction.

1. Given a graph G, find the biggest clique.

2. Given a set of integers S, divide it into two sets whose sum is equal.

Extra Credit: Problem 4 (30 points)

Prove that 2CNF-SAT is in P. **Hint:** reduce this to a decision problems for directed graphs.