15-453 Formal Languages, Automata, and Computation

Homework Assignment 9

Due Friday, April 21

Problem 1 (20 points)

Prove the following properties of the polynomial-time reducibility:

(a) If $A \leq_P B$ and $B \leq_P C$ then $A \leq_P C$.

(b) $A \leq_P A^c$ if and only if $A^c \leq_P A$.

Problem 2 (20 points)

We say that two graphs G_1 and G_2 are isomorphic if there exists a bijective function between their nodes which preserves the node incidence. Let

$$\text{GRAPHISO} = \{\langle G1, G2\rangle \mid G1 \text{ and } G2 \text{ are isomorphic}\}.$$

Prove that GRAPHISO $\in$ NP by describing a polynomial time algorithm to verify the language.

Problem 2 (30 points)

A *k-coloring* of an undirected graph is an assignment of colors $\{1, 2, \ldots, k\}$ to the nodes of the graph, such that no two adjacent vertices have the same color. Define

$$\text{k-COLOR} = \{\langle G\rangle \mid G \text{ is a graph that is k-colorable}\}.$$

Prove that 2-COLOR $\in$ P, while 3-color $\in$ NP.

Problem 4 (30 points)

Prove that the class NP is closed under union, intersection, concatenation and star.

Problem 5 (20 points extra credit)

Prove that if NP $\neq$ co-NP then P $\neq$ NP.