15-453 Formal Languages, Automata, and Computation

Homework Assignment 8

Due Wednesday, April 12

Problem 1 (40 points)

Determine the complexity classes of the problems below. In each case, give a high-level description of the algorithm used to achieve the complexity bound. The computational model is a deterministic 1-tape Turing machine.

1. $\{\langle m, n, k\rangle \mid m$, n and k are natural numbers in unary and $m + n = k\}$
2. $\{\langle m, n, k\rangle \mid m$, n and k are natural numbers in binary and $m + n = k\}$
3. $\{\langle m, n, k\rangle \mid m$, n and k are natural numbers in unary and $m * n = k\}$

Estimates that are not as close as possible will receive only partial credit.

Problem 2 (30 points)

Show that the class $\mathcal{P}$ is closed under union, concatenation and complement. [This is problem 7.6 from the textbook]

Problem 3 (30 points)

Show that the class $\mathcal{P}$ is closed under the star operation. (Hint: On input $y = y_1 \ldots y_n$ for $y_i \in \Sigma$, build a table indicating for each $i \leq j$ whether the substring $y_i \cdots y_j \in A^*$ for any $A \in P$). [This is problem 7.13 from the textbook]

Problem 4 (20 points extra credit)

Show that the language $\{\langle M, w\rangle \mid M$ accepts w in at most $|w|^2$ steps$\}$ is in $\mathcal{P}$.