15-453 Formal Languages, Automata, and Computation

Homework Assignment 6

Due Friday, March 10

Problem 1 (30 points)

For each of the following languages, determine whether it is Turing-recognizable, Turing-decidable or neither. Prove your answer.

a. $\{\langle M\rangle \mid M$ is a Turing machine which takes more than 2001 steps on input $\varepsilon\}$

b. $\{\langle M\rangle \mid M$ is a Turing machine accepting the empty string$\}$

c. $\{\langle M\rangle \mid M$ is a Turing machine accepting exactly 2001 different strings$\}$

d. $\{\langle M\rangle \mid M$ is a Turing machine accepting at least one string$\}$

e. $\{\langle M\rangle \mid M$ is a Turing machine accepting no strings at all$\}$

Problem 2 (30 points)

Show that $A_{inf} = \{\langle M\rangle \mid M$ is a DFA and $\mathcal{L}(M)$ is an infinite language $\}$ is decidable. [This is Problem 4.5 in the textbook.]

Problem 3 (40 points)

In this exercise we explore the notion of an enumerator and its relations to Turing-decidability. Informally, an enumerator is a special kind of two-tape Turing machine. Its first tape is a read/write working tape, while the second tape is an write only output tape. Both tapes are initially empty. The enumerator can move in the usual way across the working tape, but it has only two ways to access the output tape. It can successively write symbols to it, or at some point, erase the output tape completely and position itself to its beginning. Formally, an enumerator E is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{print}, q_{halt})$ where Q, Σ, Γ have their usual meaning, and $q_0, q_{print}, q_{halt} \in Q$ are the initial, print and halt states of the enumerator. The function $\delta\colon Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \times \Sigma_\varepsilon$ is a transition function where $\delta(q, a) = (r, b, D, c)$ denotes that E, upon reading a from the working tape while in the state q, writes b to the working tape and moves in direction D, writes c to the output tape and moves to the right on the output tape, and then finally switches to the state r. If $r = q_{print}$ then the output tape is reset to blank and its head moved to the beginning. E halts when it enters into state q_{halt}. The language *enumerated* by E is $\mathcal{L}(E) =$

$\{w \mid w$ appears on the output tape when q_{print} is entered$\}$. We say that E enumerates $\mathcal{L}(E)$ in lexicographical order if the words occur on the output tape in a sorted fashion. See page 141 of the textbook for more information. Prove the following properties of enumerators

- If a language is enumerated by some enumerator, then there is an "efficient" enumerator which enumerates the words without repeating them.
- A language L is decidable if and only if some enumerator enumerates it in lexicographical order.

Problem 4 (20 points extra credit)

Let C be a language. Prove that C is Turing-recognizable if and only if there exists a decidable language D such that $C = \{x \mid \exists y \text{ s. t. } \langle x, y\rangle \in D\}$. [This is problem 4.17 in the textbook.]