15-453 Formal Languages, Automata, and Computation

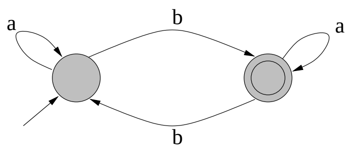
Assignment 1

Due: Friday, January 28

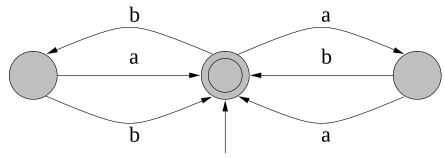
Problem 1 (15 points)

Describe the languages recognized by the following finite automata.

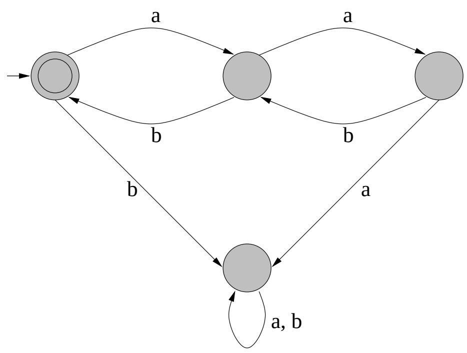
1.



2.



3.



Problem 2 (20 points)

Design finite automata to recognize the following languages over the alphabet $\Sigma = \{0, 1\}$. Present the automata by their state-graphs.

1. $L_1 = \{w \mid \text{ binary number represented by } w \text{ is divisible by } 3\}$

2. $L_2 = \{\text{Entwine}(x, y, z) \mid x + y = z\}$. The function Entwine is defined over binary numbers recursively as

$$\text{Entwine}(x, y, z) = \begin{cases} \varepsilon & \text{if } x = y = z = 0 \\ x_0 y_0 z_0 \,\text{Entwine}(x_1, y_1, z_1) & \text{otherwise} \end{cases}$$

   where $w_0 = w \bmod 2$ and $w_1 = w \operatorname{div} 2$.

   Example: The word $101\,011\,110\,001 = \text{Entwine}(101, 110, 1011) \in L_2$ since $101 + 110 = 1011$ in binary. On the other hand, $011\,111\,001 \notin L_2$ because it encodes the false statement $10 + 11 = 111$.

Problem 3 (20 points)

Prove that if $A \subseteq \Sigma^*$ is a regular language, then the language $\text{Prefix}(A) = \{w \in \Sigma^* \mid x = wy \text{ for some } x \in A \text{ and } y \in \Sigma^*\}$ is also regular.

Problem 4 (20 points)

Give a non-trivial example of a language L and two finite automata, a deterministic M_d and a nondeterministic M_n, such that L is recognized by both M_d and M_n, but M_n has a much simpler state-graph then M_d. The 5 most interesting and original examples will be awarded 5 extra points.

Problem 5 (25 points)

An arithmetic progression is the set $\{p + qn \mid n = 0, 1, 2, \ldots\}$ for some $p, q \in \mathbb{N}$. Show that if $L \subseteq \{a\}^*$ is regular, then $\{n : a^n \in L\}$ is a union of finitely many arithmetic progressions.