Weighted Low Rank Approximations with Provable Guarantees

Ilya Razenshteyn, Zhao Song, and **David P. Woodruff**

MIT, UT-Austin, IBM Almaden

IMDB

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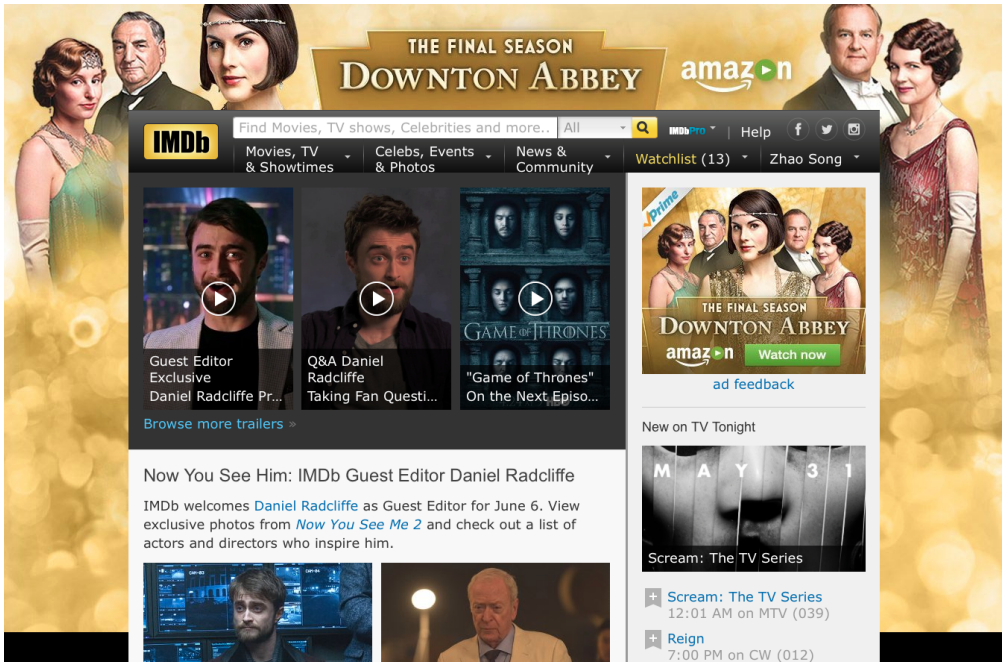
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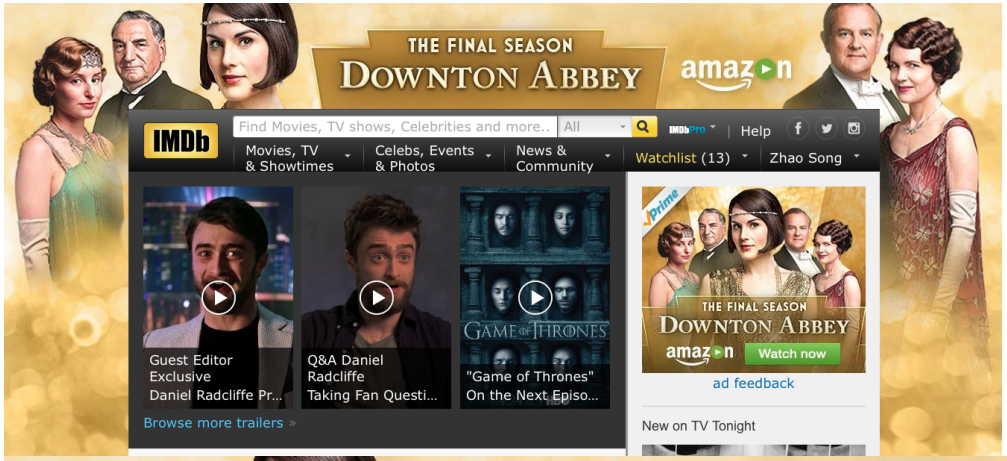

Rating Scale : 1, 2, ..., 10

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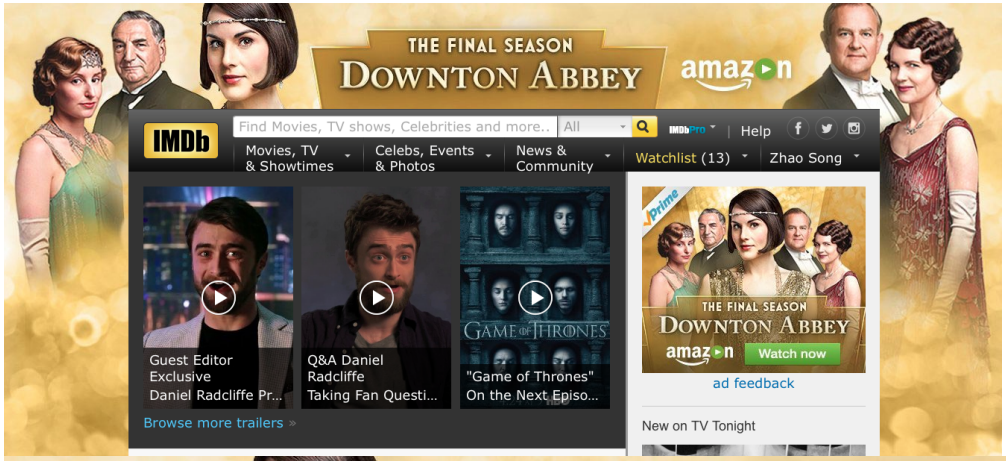
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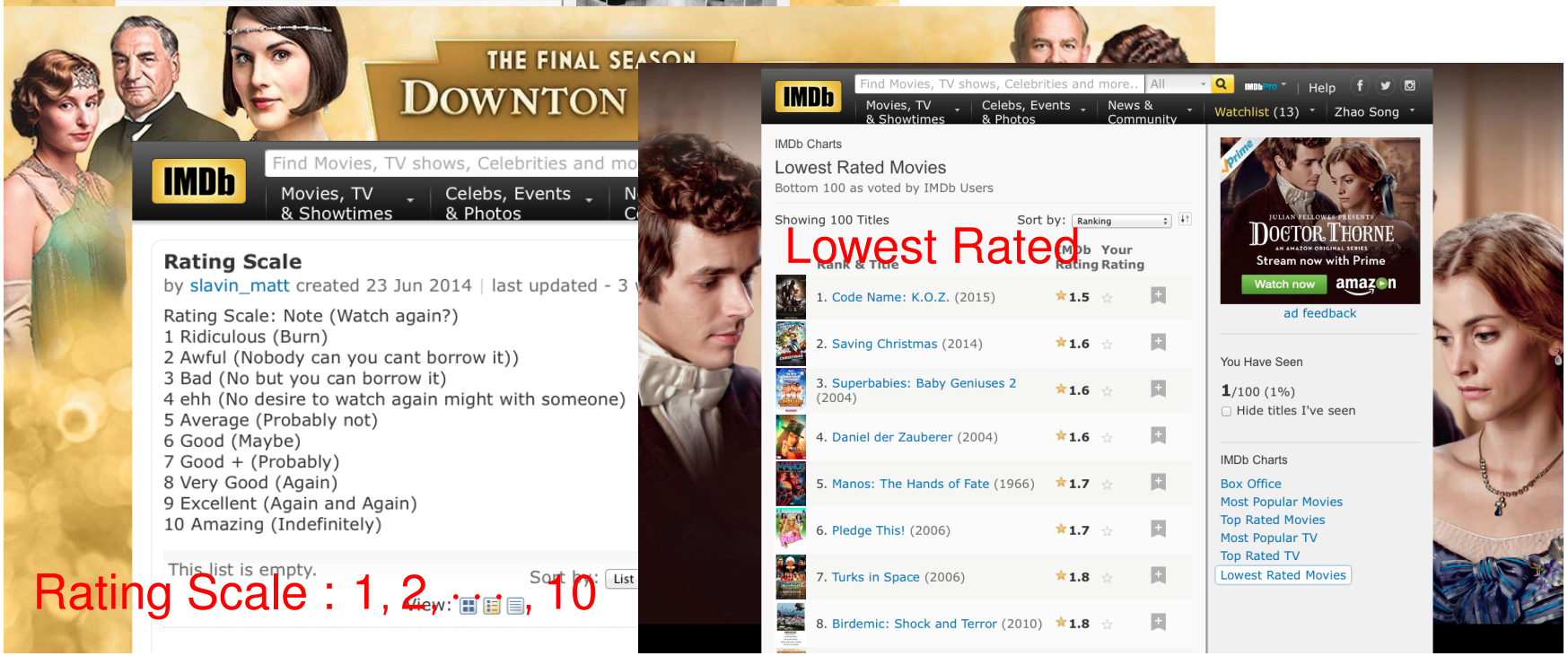
Rating Scale
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Rating Scale: Note (Watch again?)
1 Ridiculous (Burn)
2 Awful (Nobody can you cant borrow it))
3 Bad (No but you can borrow it)
4 ehh (No desire to watch again might with someone)
5 Average (Probably not)
6 Good (Maybe)
7 Good + (Probably)
8 Very Good (Again)
9 Excellent (Again and Again)
10 Amazing (Indefinitely)
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Bottom 100 as voted by IMDb Users
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3. Superbabies: Baby Geniuses 2 (2004) 1.6
4. Daniel der Zauberer (2004) 1.6
5. Manos: The Hands of Fate (1966) 1.7
6. Pledge This! (2006) 1.7
7. Turks in Space (2006) 1.8
8. Birdemic: Shock and Terror (2010) 1.8
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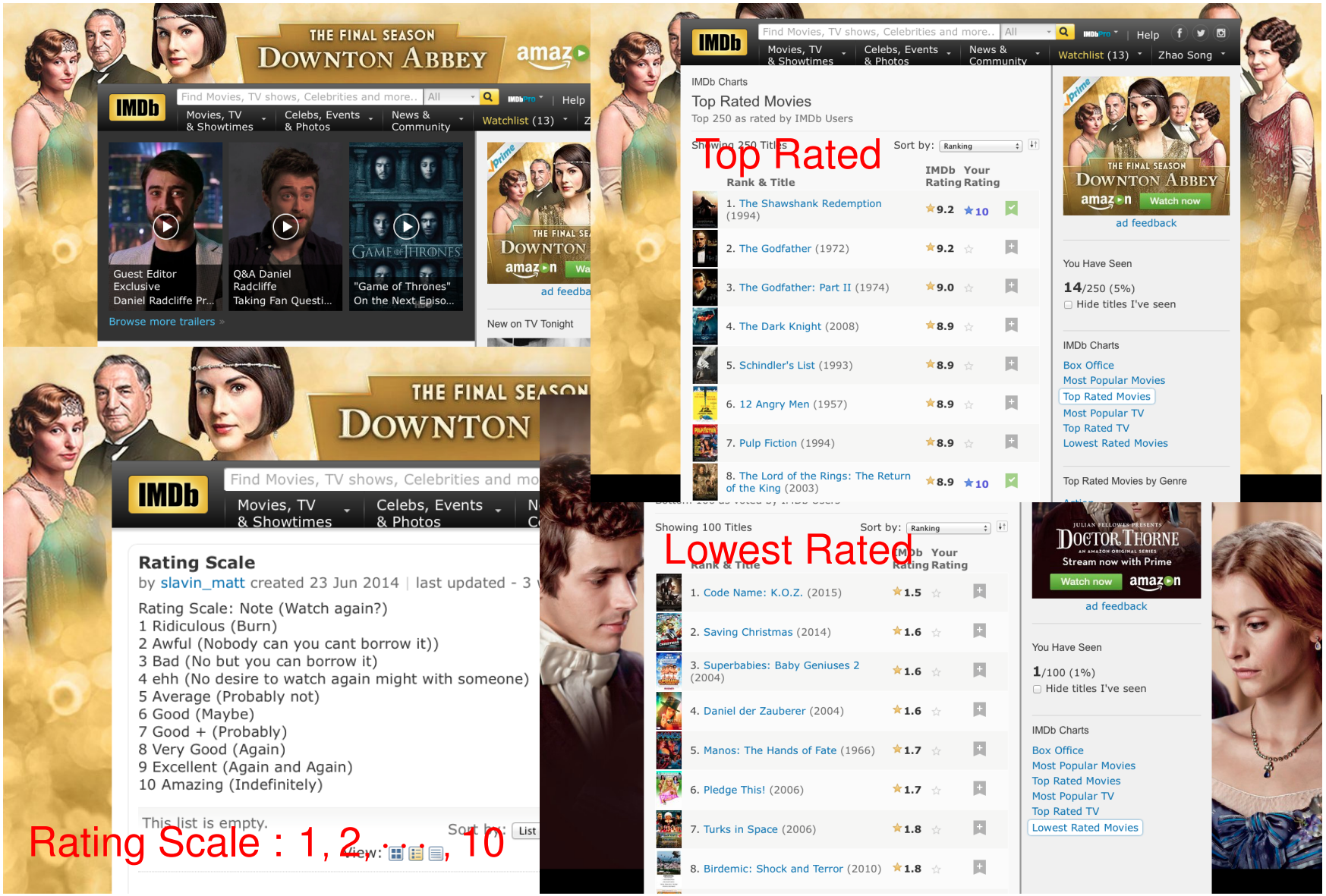
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Top Rated
IMDb Charts
Top Rated Movies
Top 250 as rated by IMDb Users
1. The Shawshank Redemption (1994) 9.2 10
2. The Godfather (1972) 9.2
3. The Godfather: Part II (1974) 9.0
4. The Dark Knight (2008) 8.9
5. Schindler's List (1993) 8.9
6. 12 Angry Men (1957) 8.9
7. Pulp Fiction (1994) 8.9
8. The Lord of the Rings: The Return of the King (2003) 8.9 10
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by slavin_matt created 23 Jun 2014
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7. Turks in Space (2006) 1.8
8. Birdemic: Shock and Terror (2010) 1.8

Motivation

Motivation

DIVIDED WE FALL
CIVIL WAR
MAY 6
JIM CARREY
MASK
FROM ZERO TO HERO
HANGOVER
LINCOLN
ZOOTOPIA
MARCH 4
Harry Potter
THE LORD OF THE RINGS

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Motivation

	2	7	7	6	9	6	4	6	6	8
	9	10	4	3	3	9	8	9	3	1
	4	3	6	5	3	7	5	3	7	1
	7	2	9	3	7	5	8	7	9	5
	5	6	5	2	10	3	8	3	9	7
	2	6	10	8	1	6	7	6	6	9
	7	1	4	6	1	5	10	7	10	4
	8	9	8	4	4	3	8	4	2	4

Motivation

	Action		Comedy		Historical		Cartoon		Magical	
	2	7	7	6	9	6	4	6	6	8
	9	10	4	3	3	9	8	9	3	1
	4	3	6	5	3	7	5	3	7	1
	7	2	9	3	7	5	8	7	9	5
	5	6	5	2	10	3	8	3	9	7
	2	6	10	8	1	6	7	6	6	9
	7	1	4	6	1	5	10	7	10	4
	8	9	8	4	4	3	8	4	2	4

Motivation

	Action		Comedy		Historical		Cartoon		Magical	
	2 $\frac{1}{5}$	7 $\frac{1}{5}$	7	6	9	6	4	6	6	8
	9 $\frac{1}{1}$	10 $\frac{1}{1}$	4	3	3	9	8	9	3	1
	4 $\frac{1}{1}$	3 $\frac{1}{1}$	6	5	3	7	5	3	7	1
	7 $\frac{1}{5}$	2 $\frac{1}{5}$	9	3	7	5	8	7	9	5
	5 $\frac{1}{1}$	6 $\frac{1}{1}$	5	2	10	3	8	3	9	7
	2 $\frac{1}{4}$	6 $\frac{1}{4}$	10	8	1	6	7	6	6	9
	7 $\frac{1}{6}$	1 $\frac{1}{6}$	4	6	1	5	10	7	10	4
	8 $\frac{1}{1}$	9 $\frac{1}{1}$	8	4	4	3	8	4	2	4

Motivation

	Action		Comedy		Historical	Cartoon	Magical
	$2\frac{1}{5}$	$7\frac{1}{5}$	$7\frac{1}{1}$	$6\frac{1}{1}$	9	6	4	6	6	8
	$9\frac{1}{1}$	$10\frac{1}{1}$	$4\frac{1}{1}$	$3\frac{1}{1}$	3	9	8	9	3	1
	$4\frac{1}{1}$	$3\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{1}$	3	7	5	3	7	1
	$7\frac{1}{5}$	$2\frac{1}{5}$	$9\frac{1}{6}$	$3\frac{1}{6}$	7	5	8	7	9	5
	$5\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{3}$	$2\frac{1}{3}$	10	3	8	3	9	7
	$2\frac{1}{4}$	$6\frac{1}{4}$	$10\frac{1}{2}$	$8\frac{1}{2}$	1	6	7	6	6	9
	$7\frac{1}{6}$	$1\frac{1}{6}$	$4\frac{1}{2}$	$6\frac{1}{2}$	1	5	10	7	10	4
	$8\frac{1}{1}$	$9\frac{1}{1}$	$8\frac{1}{4}$	$4\frac{1}{4}$	4	3	8	4	2	4

Motivation

	Action		Comedy		Historical		Cartoon	Magical	
	$2\frac{1}{5}$	$7\frac{1}{5}$	$7\frac{1}{1}$	$6\frac{1}{1}$	$9\frac{1}{3}$	$6\frac{1}{3}$	4	6	6	8
	$9\frac{1}{1}$	$10\frac{1}{1}$	$4\frac{1}{1}$	$3\frac{1}{1}$	$3\frac{1}{6}$	$9\frac{1}{6}$	8	9	3	1
	$4\frac{1}{1}$	$3\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{1}$	$3\frac{1}{4}$	$7\frac{1}{4}$	5	3	7	1
	$7\frac{1}{5}$	$2\frac{1}{5}$	$9\frac{1}{6}$	$3\frac{1}{6}$	$7\frac{1}{2}$	$5\frac{1}{2}$	8	7	9	5
	$5\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{3}$	$2\frac{1}{3}$	$10\frac{1}{7}$	$3\frac{1}{7}$	8	3	9	7
	$2\frac{1}{4}$	$6\frac{1}{4}$	$10\frac{1}{2}$	$8\frac{1}{2}$	$1\frac{1}{5}$	$6\frac{1}{5}$	7	6	6	9
	$7\frac{1}{6}$	$1\frac{1}{6}$	$4\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{4}$	$5\frac{1}{4}$	10	7	10	4
	$8\frac{1}{1}$	$9\frac{1}{1}$	$8\frac{1}{4}$	$4\frac{1}{4}$	$4\frac{1}{1}$	$3\frac{1}{1}$	8	4	2	4

Motivation

	Action		Comedy		Historical		Cartoon		Magical	
	2 $\frac{1}{5}$	7 $\frac{1}{5}$	7 $\frac{1}{1}$	6 $\frac{1}{1}$	9 $\frac{1}{3}$	6 $\frac{1}{3}$	4 $\frac{1}{2}$	6 $\frac{1}{2}$	6	8
	9 $\frac{1}{1}$	10 $\frac{1}{1}$	4 $\frac{1}{1}$	3 $\frac{1}{1}$	3 $\frac{1}{6}$	9 $\frac{1}{6}$	8 $\frac{1}{1}$	9 $\frac{1}{1}$	3	1
	4 $\frac{1}{1}$	3 $\frac{1}{1}$	6 $\frac{1}{1}$	5 $\frac{1}{1}$	3 $\frac{1}{4}$	7 $\frac{1}{4}$	5 $\frac{1}{2}$	3 $\frac{1}{2}$	7	1
	7 $\frac{1}{5}$	2 $\frac{1}{5}$	9 $\frac{1}{6}$	3 $\frac{1}{6}$	7 $\frac{1}{2}$	5 $\frac{1}{2}$	8 $\frac{1}{1}$	7 $\frac{1}{1}$	9	5
	5 $\frac{1}{1}$	6 $\frac{1}{1}$	5 $\frac{1}{3}$	2 $\frac{1}{3}$	10 $\frac{1}{7}$	3 $\frac{1}{7}$	8 $\frac{1}{5}$	3 $\frac{1}{5}$	9	7
	2 $\frac{1}{4}$	6 $\frac{1}{4}$	10 $\frac{1}{2}$	8 $\frac{1}{2}$	1 $\frac{1}{5}$	6 $\frac{1}{5}$	7 $\frac{1}{1}$	6 $\frac{1}{1}$	6	9
	7 $\frac{1}{6}$	1 $\frac{1}{6}$	4 $\frac{1}{2}$	6 $\frac{1}{2}$	1 $\frac{1}{4}$	5 $\frac{1}{4}$	10 $\frac{1}{3}$	7 $\frac{1}{3}$	10	4
	8 $\frac{1}{1}$	9 $\frac{1}{1}$	8 $\frac{1}{4}$	4 $\frac{1}{4}$	4 $\frac{1}{1}$	3 $\frac{1}{1}$	8 $\frac{1}{4}$	4 $\frac{1}{4}$	2	4

Motivation

	Action		Comedy		Historical		Cartoon		Magical	
	$2\frac{1}{5}$	$7\frac{1}{5}$	$7\frac{1}{1}$	$6\frac{1}{1}$	$9\frac{1}{3}$	$6\frac{1}{3}$	$4\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$	$8\frac{1}{2}$
	$9\frac{1}{1}$	$10\frac{1}{1}$	$4\frac{1}{1}$	$3\frac{1}{1}$	$3\frac{1}{6}$	$9\frac{1}{6}$	$8\frac{1}{1}$	$9\frac{1}{1}$	$3\frac{1}{2}$	$1\frac{1}{2}$
	$4\frac{1}{1}$	$3\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{1}$	$3\frac{1}{4}$	$7\frac{1}{4}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$7\frac{1}{6}$	$1\frac{1}{6}$
	$7\frac{1}{5}$	$2\frac{1}{5}$	$9\frac{1}{6}$	$3\frac{1}{6}$	$7\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{1}$	$7\frac{1}{1}$	$9\frac{1}{4}$	$5\frac{1}{4}$
	$5\frac{1}{1}$	$6\frac{1}{1}$	$5\frac{1}{3}$	$2\frac{1}{3}$	$10\frac{1}{7}$	$3\frac{1}{7}$	$8\frac{1}{5}$	$3\frac{1}{5}$	$9\frac{1}{2}$	$7\frac{1}{2}$
	$2\frac{1}{4}$	$6\frac{1}{4}$	$10\frac{1}{2}$	$8\frac{1}{2}$	$1\frac{1}{5}$	$6\frac{1}{5}$	$7\frac{1}{1}$	$6\frac{1}{1}$	$6\frac{1}{3}$	$9\frac{1}{3}$
	$7\frac{1}{6}$	$1\frac{1}{6}$	$4\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{4}$	$5\frac{1}{4}$	$10\frac{1}{3}$	$7\frac{1}{3}$	$10\frac{1}{6}$	$4\frac{1}{6}$
	$8\frac{1}{1}$	$9\frac{1}{1}$	$8\frac{1}{4}$	$4\frac{1}{4}$	$4\frac{1}{1}$	$3\frac{1}{1}$	$8\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{1}{2}$

Matrix Completion

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Output : $A_{\bar{\Omega}}$ s.t. $\text{rank}(A) = k$

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Output : $A_{\bar{\Omega}}$ s.t. $\text{rank}(A) = k$

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Output : $A_{\bar{\Omega}}$ s.t. $\text{rank}(A) = k$

Algorithm : Candes-Recht'09, Candes-Recht'10, Keshavan'12
Hardt-Wootters'14, Jain-Netrapalli-Sanghavi'13
Hardt'15, Sun-Luo'15

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Output : $A_{\bar{\Omega}}$ s.t. $\text{rank}(A) = k$

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Hardt-Wootters'14, Jain-Netrapalli-Sanghavi'13
Hardt'15, Sun-Luo'15

Hardness : Peeters'96, Gillis-Glineur'11
Hardt-Meka-Raghavendra-Weitz'14

Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n] \times [n]$ and $k \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}, k = 2$$

Output : $A_{\bar{\Omega}}$ s.t. $\text{rank}(A) = k$

Algorithm : Candes-Recht'09, Candes-Recht'10, Keshavan'12
Hardt-Wootters'14, Jain-Netrapalli-Sanghavi'13
Hardt'15, Sun-Luo'15

Hardness : Peeters'96, Gillis-Glineur'11
Hardt-Meka-Raghavendra-Weitz'14

Weighted : Srebro-Jaakkola'03, Li-Liang-Risteski'16

Weighted Matrix Completion

Weighted Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n]^2$

$A \in \{\mathbb{R}\}^{n^2}$ $W \in \{0, 1\}^{n^2}$

Weighted Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}, \Omega \subset [n]^2$
$k \in \mathbb{N}$

$A \in \{\mathbb{R}\}^{n^2}$ $W \in \{0, 1\}^{n^2}$
$k \in \mathbb{N}, r \in \mathbb{N}$

Weighted Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}, \Omega \subset [n]^2$
$k \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}$$

$A \in \{\mathbb{R}\}^{n^2}$ $W \in \{0, 1\}^{n^2}$
$k \in \mathbb{N}, r \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 1 & 0 & 9 \\ 6 & 0 & 1 & 4 \\ 0 & 5 & 3 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Weighted Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}, \Omega \subset [n]^2$
$k \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}$$

$k = 2$

$A \in \{\mathbb{R}\}^{n^2}$ $W \in \{0, 1\}^{n^2}$
$k \in \mathbb{N}, r \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 1 & 0 & 9 \\ 6 & 0 & 1 & 4 \\ 0 & 5 & 3 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$k = 2, r = 2$

Weighted Matrix Completion

Given : $A \in \{\mathbb{R}, ?\}^{n^2}$, $\Omega \subset [n]^2$
$k \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & ? & ? & 0 \\ ? & 1 & 0 & ? \\ ? & 0 & 1 & ? \\ 0 & ? & ? & 1 \end{bmatrix}$$

$$k = 2$$

Output : $A_{\bar{\Omega}}$
s.t. $\text{rank}(A) = k$

$A \in \{\mathbb{R}\}^{n^2}$ $W \in \{0, 1\}^{n^2}$
$k \in \mathbb{N}, r \in \mathbb{N}$

$$A = \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 1 & 0 & 9 \\ 6 & 0 & 1 & 4 \\ 0 & 5 & 3 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$k = 2, r = 2$$

B
s.t. $\text{rank}(B) = k$
$\|W \circ (A - B)\|_F^2 = 0$

Main Results - Summary

- Algorithm for Weighted low rank approximation(WLRA) problem

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- Algorithm for Weighted low rank approximation(WLRA) problem
  - W has r distinct rows and columns

$$W = \begin{bmatrix} 1 & 1 & 1 & 3 & 3 & 4 \\ 1 & 1 & 1 & 3 & 3 & 4 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 7 & 7 & 7 & 4 & 4 & 6 \end{bmatrix}, \; r = 3$$

Main Results - Summary

- Algorithm for Weighted low rank approximation(WLRA) problem
  - W has r distinct rows and columns
  - W has r distinct columns

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 & 3 \\ 3 & 3 & 3 & 1 & 1 & 5 \\ 4 & 4 & 4 & 1 & 1 & 1 \\ 5 & 5 & 5 & 1 & 1 & 3 \\ 6 & 6 & 6 & 1 & 1 & 5 \end{bmatrix}, \; r = 3$$

Main Results - Summary

- Algorithm for Weighted low rank approximation(WLRA) problem
  - W has r distinct rows and columns
  - W has r distinct columns
  - W has rank at most r

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix}, r = 3$$

Main Results - Summary

- Algorithm for Weighted low rank approximation(WLRA) problem
  - W has r distinct rows and columns
  - W has r distinct columns
  - W has rank at most r
- Hardness for Weighted low rank approximation(WLRA) problem

r Distinct Rows and Columns

r Distinct Rows and Columns

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

r Distinct Rows and Columns

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

$$W = \begin{bmatrix} 1 & 1 & 1 & 3 & 3 & 4 \\ 1 & 1 & 1 & 3 & 3 & 4 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 2 & 2 & 2 & 5 & 5 & 0 \\ 7 & 7 & 7 & 4 & 4 & 6 \end{bmatrix}, r = 3$$

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Output : rank-k B $\in \mathbb{R}^{n^2}$ s.t.

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Output : rank-k $B \in \mathbb{R}^{n^2}$ s.t.

$$\| W \circ (A - B) \|_F^2 \leqslant (1 + \epsilon) \min_{\text{rank}-k\ A'} \| W \circ (A - A') \|_F^2$$

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Output : rank-k B $\in \mathbb{R}^{n^2}$ s.t.
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with prob. $9/10$

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in $O((\mathrm{nnz}(A) + \mathrm{nnz}(W)) \cdot n^{\gamma}) + n \cdot 2^{\widetilde{O}(k^2 r/\epsilon)}$ time

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fixed parameter tractable

r Distinct Columns

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fixed parameter tractable

Rank r

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$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix}, r = 3$$

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in $n^{O(k^2 r/\epsilon)}$ time
previously only r=1 was known to be in polynomial time

Main Results - Hardness

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[Feige'02, Goerdt-Lanka'04]

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Random-4SAT Hypothesis

Main Results - Hardness

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Random-4SAT Hypothesis

Given : random 4-SAT formula $\mathcal{S}$ on n variables

Main Results - Hardness

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Given : random 4-SAT formula $\mathcal{S}$ on n variables
each clause has 4 literals

S_1 $(x_4 \vee x_{32} \vee x_9 \vee x_{20})$

Main Results - Hardness

[Feige'02, Goerdt-Lanka'04]

Random-4SAT Hypothesis

Given : random 4-SAT formula $\mathcal{S}$ on n variables
each clause has 4 literals
each of $\Theta(n^4)$ clauses is picked ind. with prob. $\Theta(1/n^3)$

$$S_1 \qquad (x_4 \vee x_{32} \vee x_9 \vee x_{20})$$
$$S_2 \qquad (x_2 \vee x_{13} \vee x_5 \vee x_{25})$$
$$\cdots\cdots$$
$$S_{\Theta(n)} \qquad (x_{11} \vee x_{19} \vee x_{24} \vee x_6)$$

$$\mathcal{S} = S_1 \wedge S_2 \wedge \cdots \wedge S_{\Theta(n)}$$

Main Results - Hardness

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Any algorithm that

$$
\begin{array}{ll}
S_1 & (x_4 \vee x_{32} \vee x_9 \vee x_{20}) \\
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 & \cdots\cdots \\
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\end{array}
$$

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Main Results - Hardness

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Any algorithm that
outputs 1 with prob. 1 when $\mathcal{S}$ is satisfiable

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outputs 0 with prob. $\geqslant 1/2$ when $\mathcal{S}$ is unsatisfiable

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Main Results - Hardness

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Requires : $2^{\Omega(n)}$ time

$$S_1 \quad (x_4 \vee x_{32} \vee x_9 \vee x_{20})$$
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Main Results - Hardness

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Weighted Low Rank Approximation Hardness

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Given : $A \in \mathbb{R}^{n^2}$, distinct r columns $W \in \mathbb{R}^{n^2}$

Main Results - Hardness

Weighted Low Rank Approximation Hardness

Given : $A \in \mathbb{R}^{n^2}$, distinct r columns $W \in \mathbb{R}^{n^2}$
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Output : rank-k B s.t. $\|W \circ (A - B)\|_F^2 \leqslant (1 + \epsilon)\,\text{OPT}$
with prob. $9/10$

Main Results - Hardness

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Assume : Random-4SAT

Main Results - Hardness

Weighted Low Rank Approximation Hardness

Given : $A \in \mathbb{R}^{n^2}$, distinct r columns $W \in \mathbb{R}^{n^2}$
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Assume : Random-4SAT

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Requires : $2^{\Omega(r)}$ time !

Outline of Technique

- Obtain a lower bound on the cost

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Lower Bound on the Cost

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[Jeronimo-Perrucci-Tsigaridas'13]

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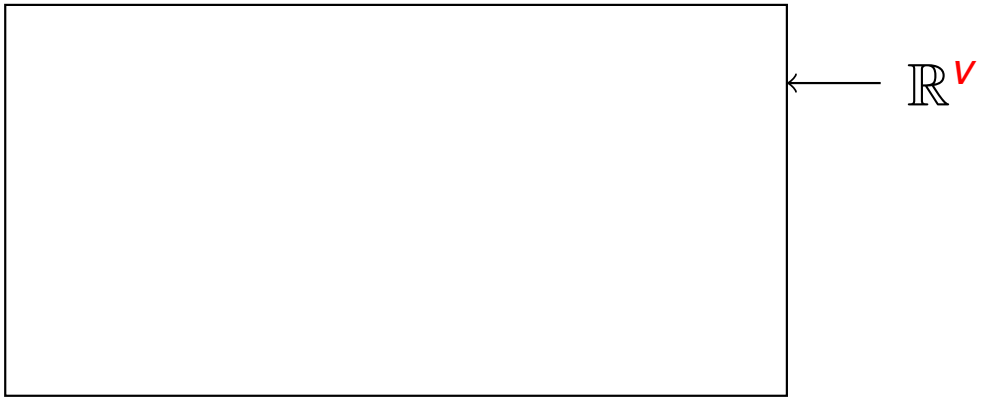

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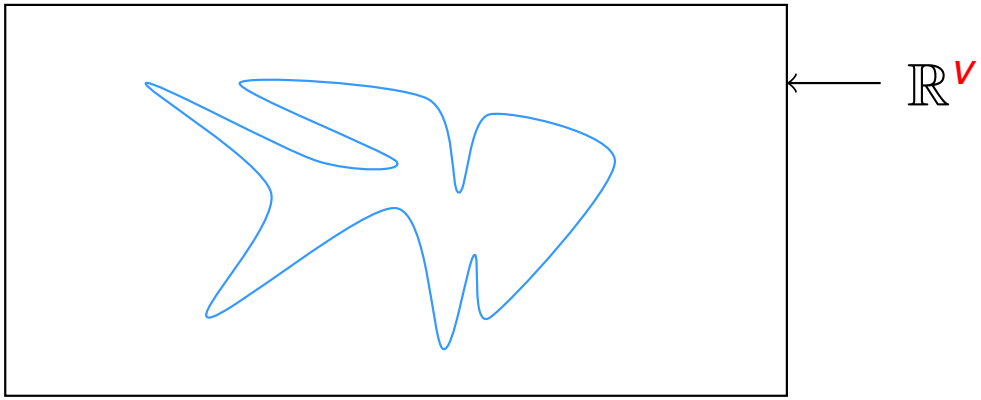

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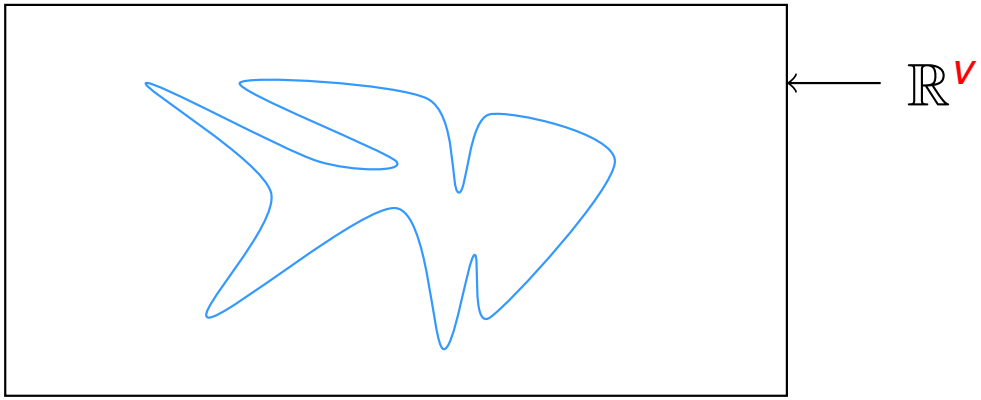
minimum value that nonnegative g takes over T is either 0 or $\geqslant (H + m)^{-d^v}$



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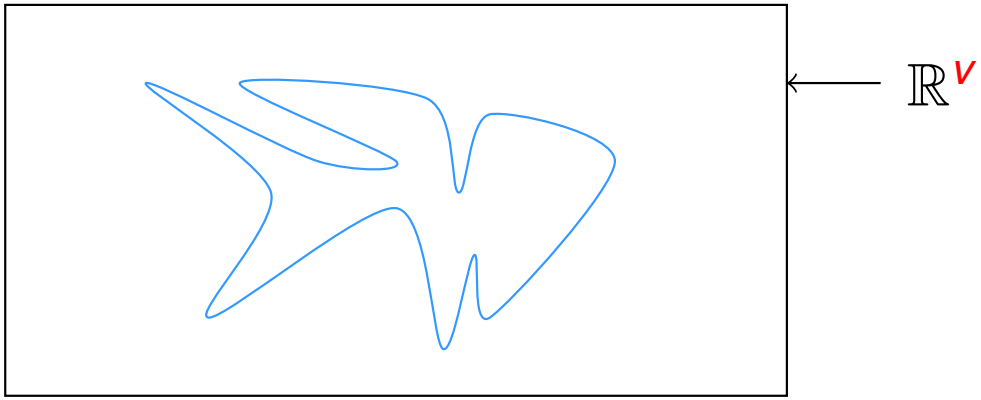

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It takes $(md)^{O(v)}$ poly$(\log H)$ time to decide if $\exists$ a solution to polynomial system P

Multiple Regression Sketch

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without blowing up degree and #constraints too much

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Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix}$$

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$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \quad D_{W_1} = \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \quad D_{W_2} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & \boxed{1} & 2 & 2 & 2 \\ 0 & 1 & \boxed{2} & 1 & 2 & 3 \\ 1 & 1 & \boxed{3} & 2 & 4 & 4 \\ 0 & 0 & \boxed{4} & 0 & 4 & 4 \\ 1 & 0 & \boxed{5} & 1 & 6 & 5 \\ 0 & 0 & \boxed{6} & 0 & 6 & 6 \end{bmatrix} \quad D_{W_3} = \begin{bmatrix} 1 & & & & & \\ & 2 & & & & \\ & & 3 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \quad D_{W_4} = \begin{bmatrix} 2 & & & & & \\ & 1 & & & & \\ & & 2 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \quad D_{W_5} = \begin{bmatrix} 2 & & & & & \\ & 2 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 6 & \\ & & & & & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be jth column of W
D_{W_j} be diagonal matrix with vector W_j

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & \boxed{2} \\ 0 & 1 & 2 & 1 & 2 & \boxed{3} \\ 1 & 1 & 3 & 2 & 4 & \boxed{4} \\ 0 & 0 & 4 & 0 & 4 & \boxed{4} \\ 1 & 0 & 5 & 1 & 6 & \boxed{5} \\ 0 & 0 & 6 & 0 & 6 & \boxed{6} \end{bmatrix} \quad D_{W_6} = \begin{bmatrix} 2 & & & & & \\ & 3 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2$

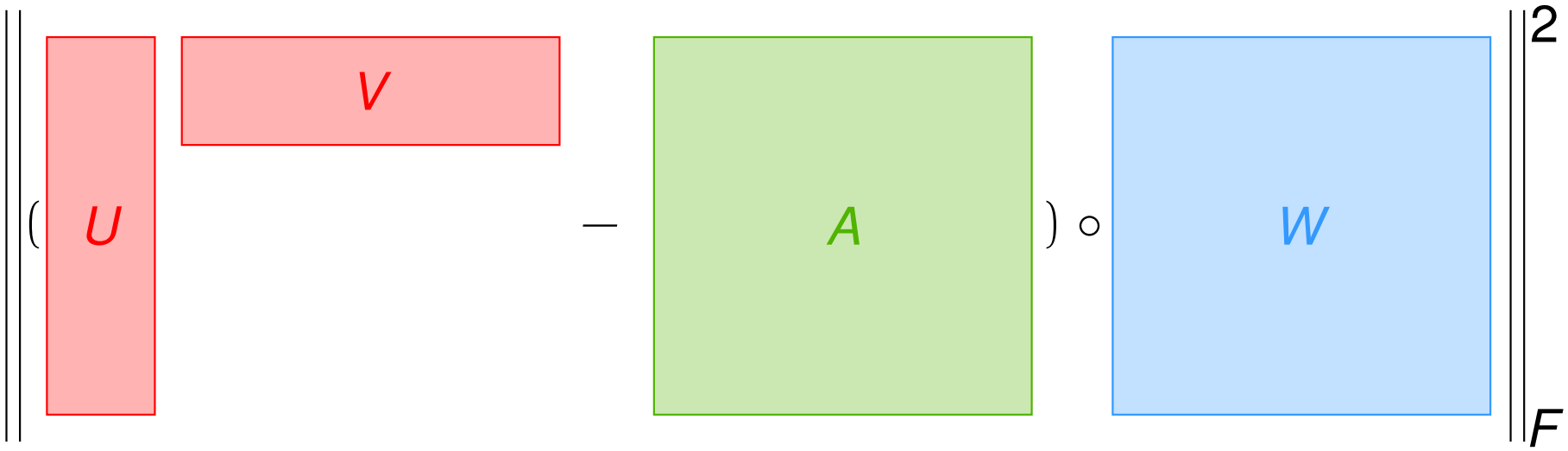
$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $= \|(UV - A) \circ W\|_F^2$



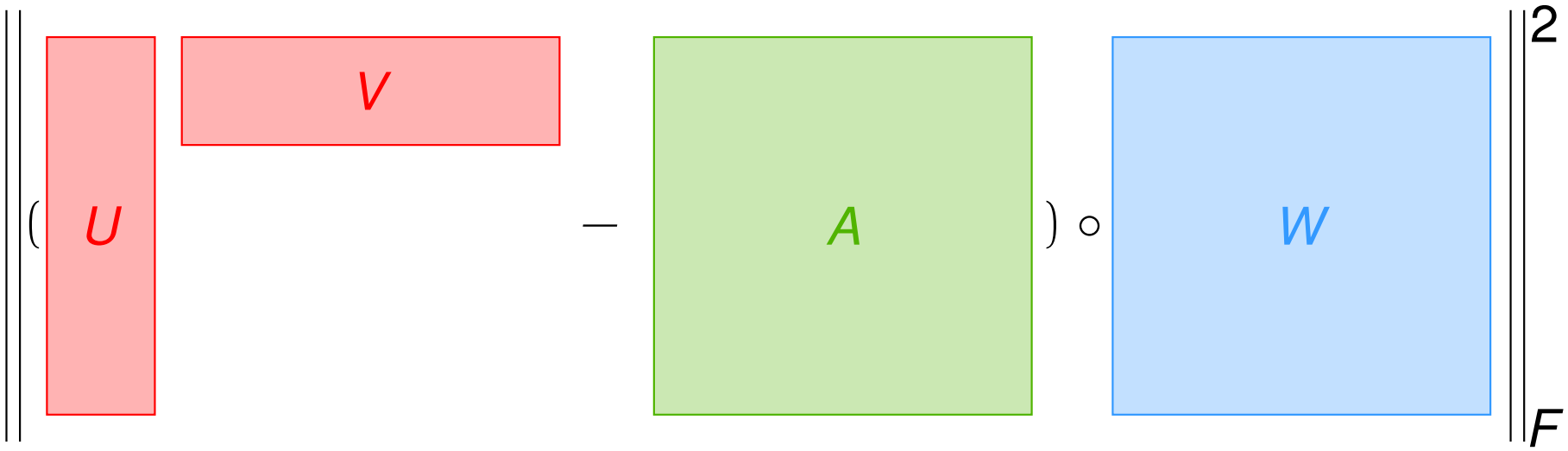
Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $= \|(UV - A) \circ W\|_F^2$

$\sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$



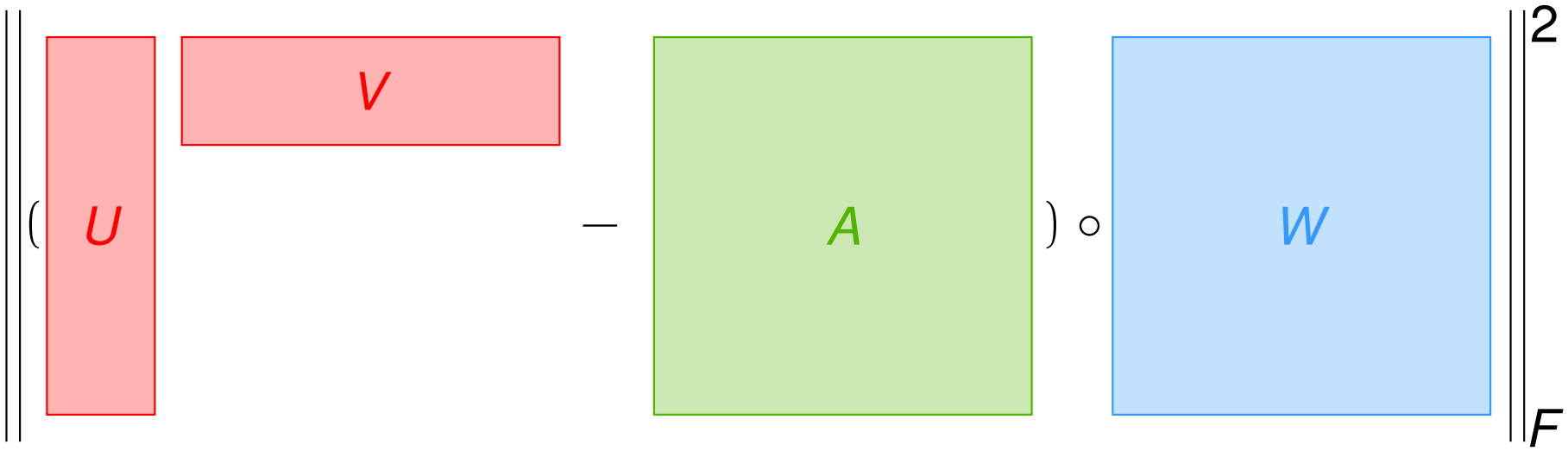
Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $= \|(UV - A) \circ W\|_F^2 =$

$\sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$



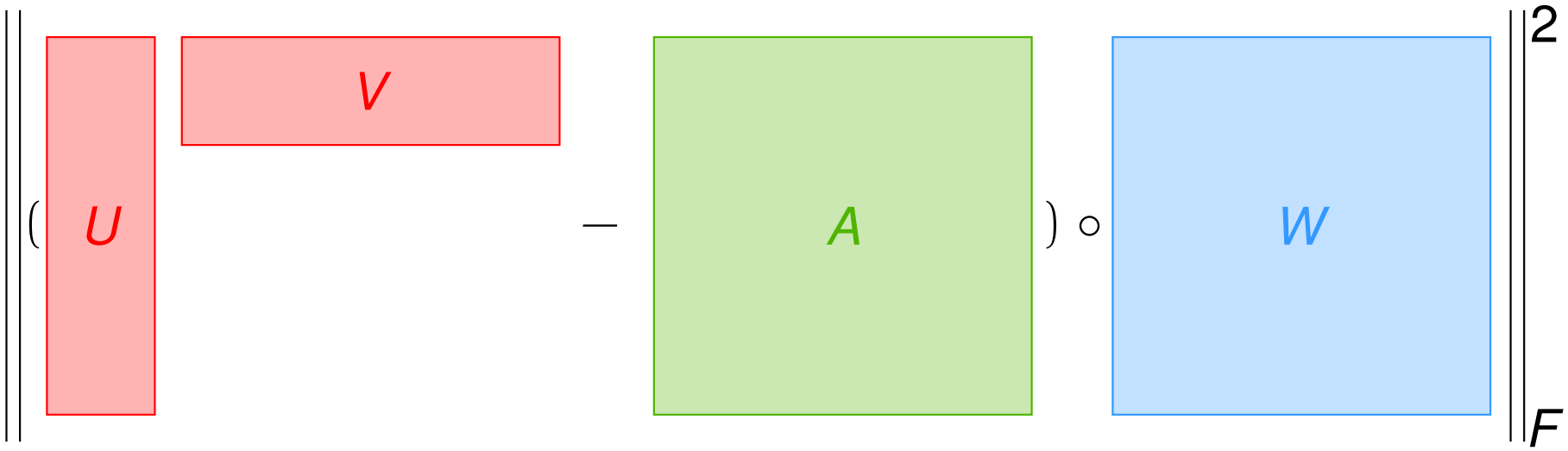
Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm :

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 = \|(UV - A) \circ W\|_F^2 = \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

Guess a Sketch

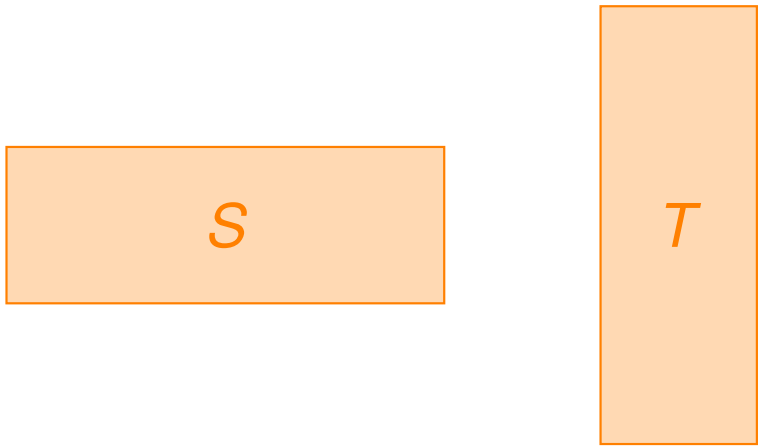
Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 = \|(UV - A) \circ W\|_F^2 = \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_1} U = \boxed{S} \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} \boxed{U}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_2} U = \boxed{S} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \boxed{U}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_3} U = \boxed{S} \begin{bmatrix} 1 & & & & & \\ & 2 & & & & \\ & & 3 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix} \boxed{U}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} UV_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_4} U = \boxed{S} \begin{bmatrix} 2 & & & & & \\ & 1 & & & & \\ & & 2 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} \boxed{U}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_5} U = \boxed{S} \begin{bmatrix} 2 & & & & & \\ & 2 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 6 & \\ & & & & & 6 \end{bmatrix} \boxed{U}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

$$S D_{W_6} U = S \begin{bmatrix} 2 & & & & & \\ & 3 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix} U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

Create $t \times k$ variables for each of n $S D_{W_j} U$ s?

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^{n} \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^{n} \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^{n} \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

Create $t \times k$ variables for each of n $S D_{W_j} U$ s?

$$n \times t \times k$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

Create $t \times k$ variables for each of n $S D_{W_j} U$ s?

$$? \times t \times k$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$

Algorithm :

$$= \|(UV - A) \circ W\|_F^2 =$$

$$\sum_{j=1}^n \|D_{W_j} U V_j - D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} - A^i D_{W^i}\|_2^2$$

Sketch by Gaussian S, $T^\top \in \mathbb{R}^{t \times n}$

$$\sum_{j=1}^n \|S D_{W_j} U V_j - S D_{W_j} A_j\|_2^2 \qquad \sum_{i=1}^n \|U^i V D_{W^i} T - A^i D_{W^i} T\|_2^2$$

Guess $S D_{W_j} U \in \mathbb{R}^{t \times k}$ and $V D_{W^i} T \in \mathbb{R}^{k \times t}$

Create $t \times k$ variables for each of n $S D_{W_j} U$ s?

$$r \times t \times k$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \\ 0 & 0 & 4 \\ 1 & 0 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_1 = W_1$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_2 = W_2$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_3 = W_3$$

$$W = \begin{bmatrix} 1 & 1 & \boxed{1} & 2 & 2 & 2 \\ 0 & 1 & \boxed{2} & 1 & 2 & 3 \\ 1 & 1 & \boxed{3} & 2 & 4 & 4 \\ 0 & 0 & \boxed{4} & 0 & 4 & 4 \\ 1 & 0 & \boxed{5} & 1 & 6 & 5 \\ 0 & 0 & \boxed{6} & 0 & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_4 = W_1 + W_2$$

$$W = \begin{bmatrix} 1 & 1 & 1 & \boxed{2} & 2 & 2 \\ 0 & 1 & 2 & \boxed{1} & 2 & 3 \\ 1 & 1 & 3 & \boxed{2} & 4 & 4 \\ 0 & 0 & 4 & \boxed{0} & 4 & 4 \\ 1 & 0 & 5 & \boxed{1} & 6 & 5 \\ 0 & 0 & 6 & \boxed{0} & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_5 = W_1 + W_3$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 \\ 1 & 0 & 5 & 1 & 6 & 5 \\ 0 & 0 & 6 & 0 & 6 & 6 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : W_j be ith column of W

$$W_6 = W_2 + W_3$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & \boxed{2} \\ 0 & 1 & 2 & 1 & 2 & \boxed{3} \\ 1 & 1 & 3 & 2 & 4 & \boxed{4} \\ 0 & 0 & 4 & 0 & 4 & \boxed{4} \\ 1 & 0 & 5 & 1 & 6 & \boxed{5} \\ 0 & 0 & 6 & 0 & 6 & \boxed{6} \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 3 \\ 4 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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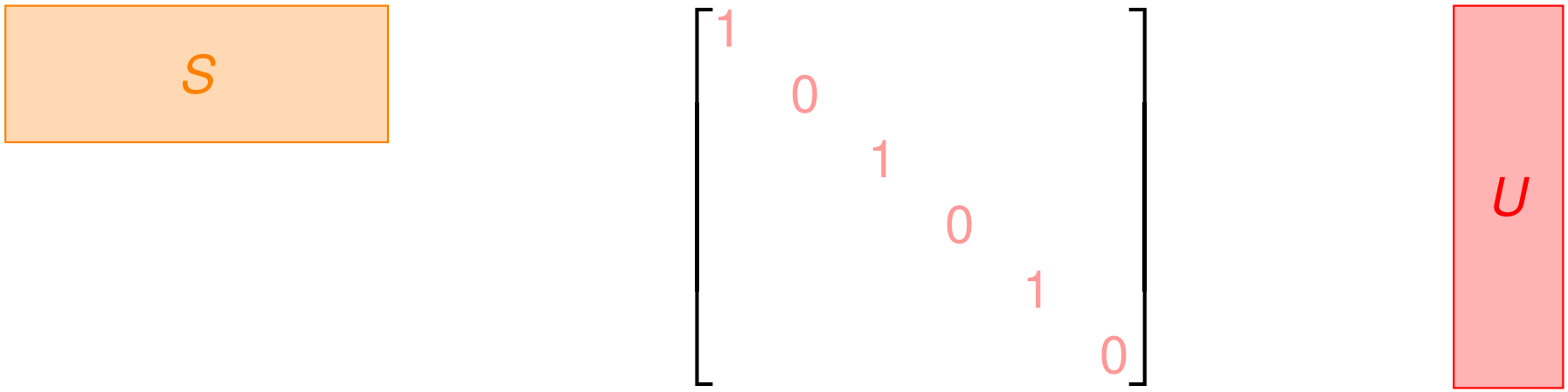
sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

create $t \times k$ variables for $SD_{W_1} U$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

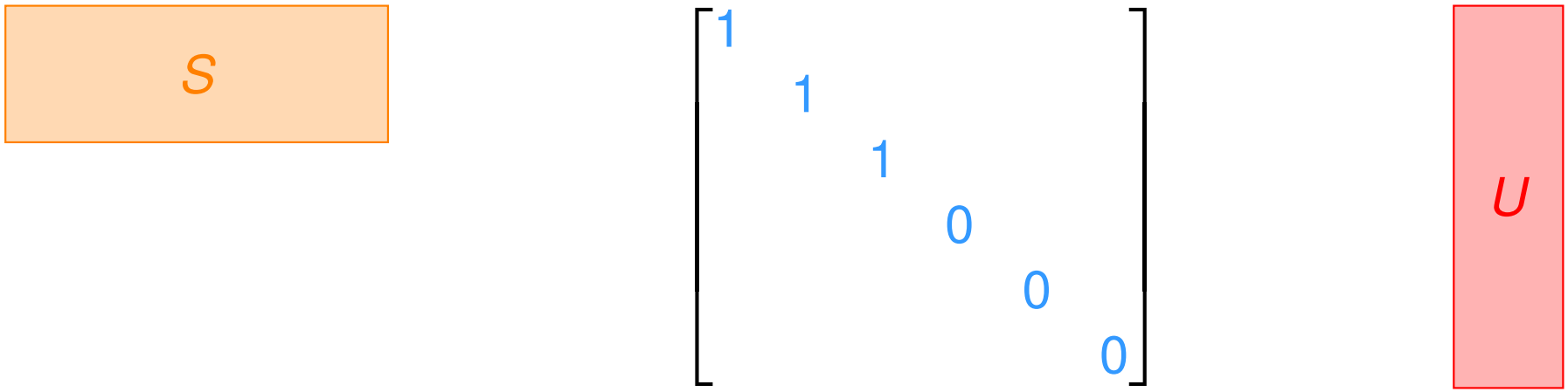
Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

create $t \times k$ variables for $SD_{W_2} U$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

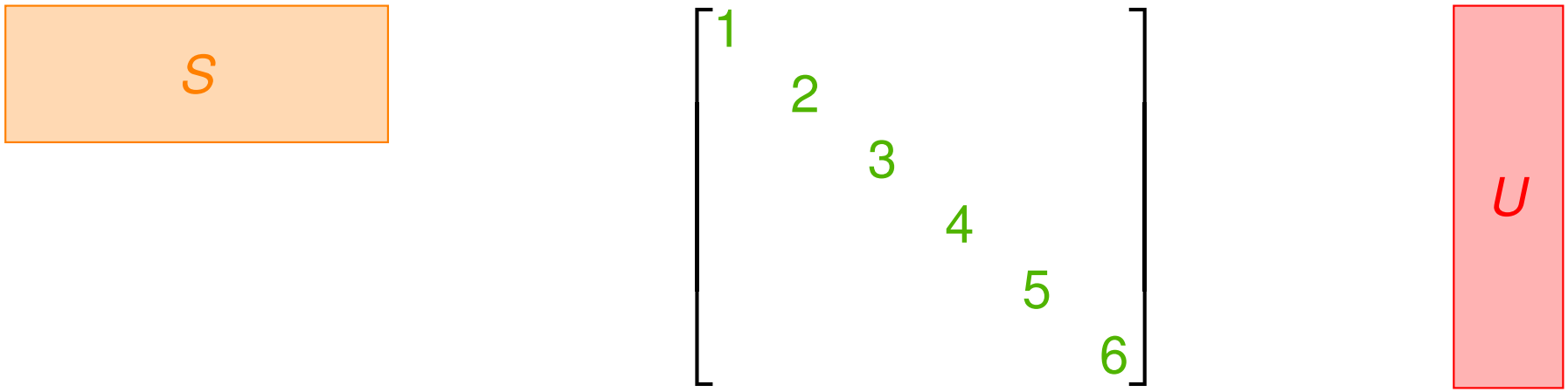
Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

create $t \times k$ variables for $SD_{W_3} U$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

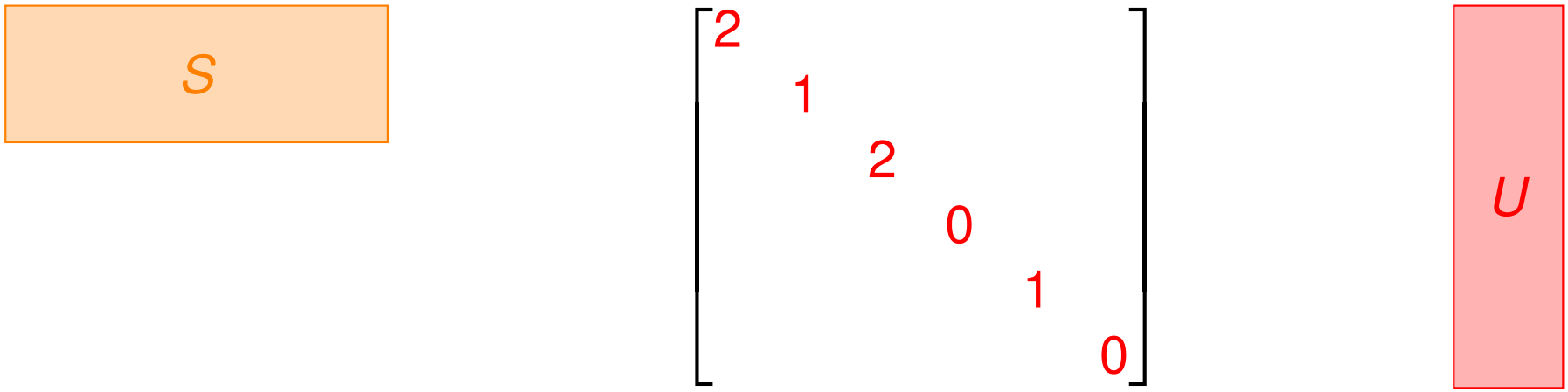
Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

write $SD_{W_4} U$ as $SD_{W_1} U + SD_{W_2} U$



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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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create variables for $SD_{W_j} U, \forall i \in [r]$

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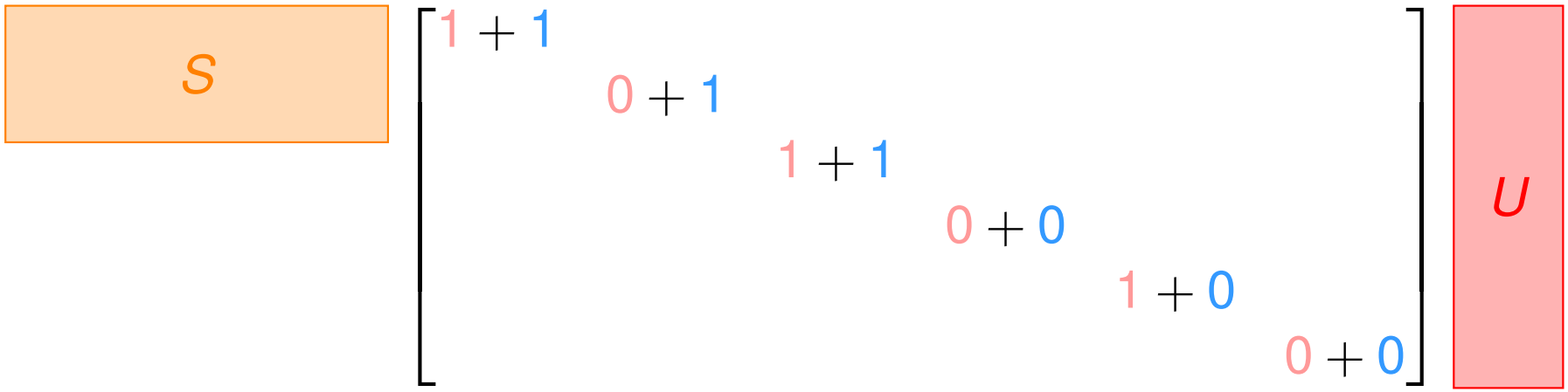

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sketch $\sum_{j=1}^{n} \|SD_{W_j} UV_j - SD_{W_j} A_j\|_2^2$

create variables for $SD_{W_j} U, \forall i \in [r]$

write $SD_{W_4} U$ as $SD_{W_1} U + SD_{W_2} U$

$$S \left(\begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \right) U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

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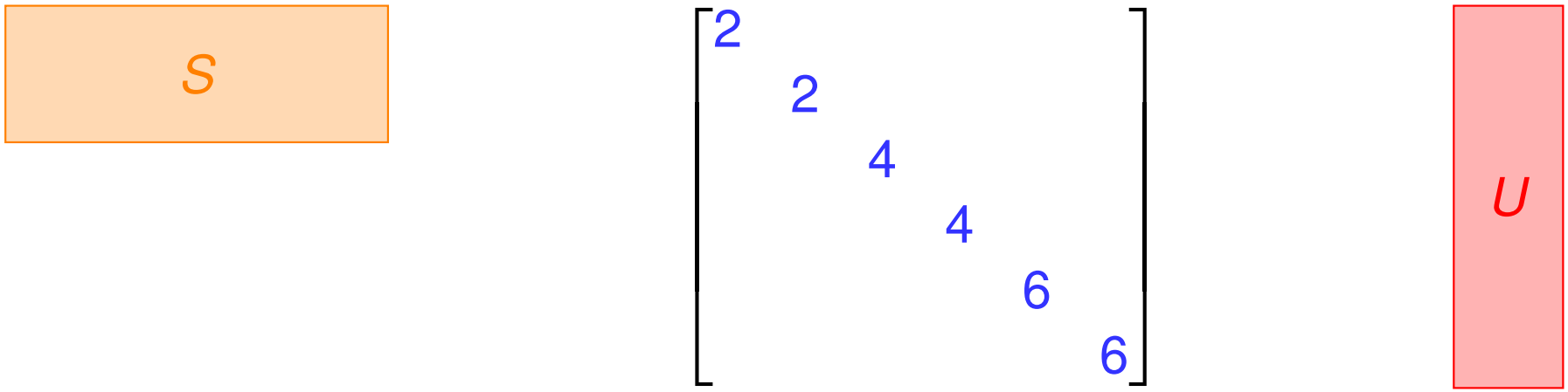
create variables for $SD_{W_j} U, \forall i \in [r]$

write $SD_{W_5} U$ as $SD_{W_1} U + SD_{W_3} U$



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create variables for $SD_{W_j} U, \forall i \in [r]$

write $SD_{W_5} U$ as $SD_{W_1} U + SD_{W_3} U$

$$S \begin{bmatrix} 1+1 & & & & & \\ & 0+2 & & & & \\ & & 1+3 & & & \\ & & & 0+4 & & \\ & & & & 1+5 & \\ & & & & & 0+6 \end{bmatrix} U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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write $SD_{W_5} U$ as $SD_{W_1} U + SD_{W_3} U$

$$S \left(\begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} 1 & & & & & \\ & 2 & & & & \\ & & 3 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix} \right) U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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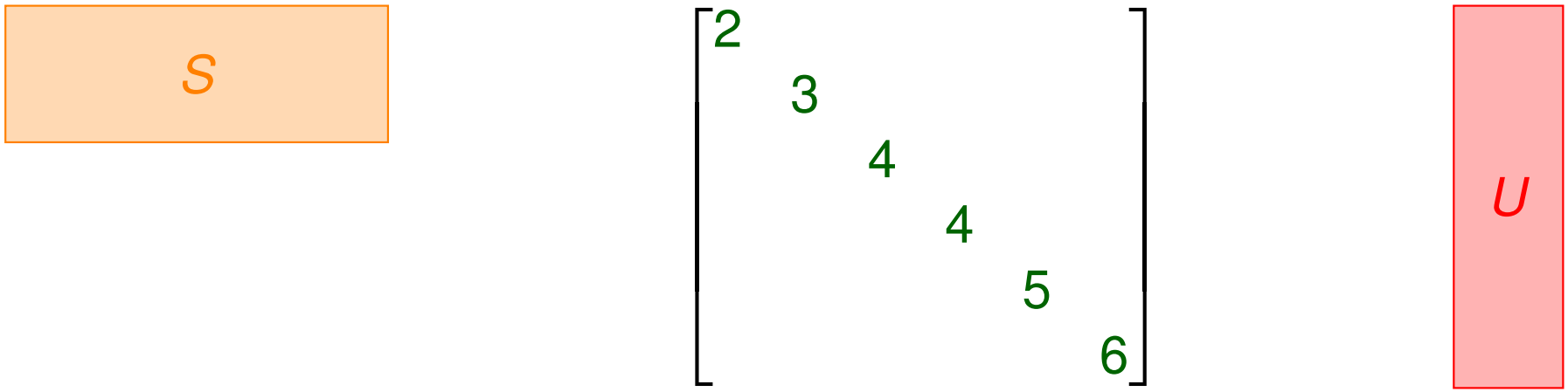

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

Algorithm : $\|(UV - A) \circ W\|_F^2 = \sum_{j=1}^{n} \|D_{W_j} UV_j - D_{W_j} A_j\|_2^2$

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$$S \begin{bmatrix} 1+1 & & & & & \\ & 1+2 & & & & \\ & & 1+3 & & & \\ & & & 0+4 & & \\ & & & & 0+5 & \\ & & & & & 0+6 \end{bmatrix} U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption :

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$$S \left(\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} 1 & & & & & \\ & 2 & & & & \\ & & 3 & & & \\ & & & 4 & & \\ & & & & 5 & \\ & & & & & 6 \end{bmatrix} \right) U$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$,

Algorithm :

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$,

Algorithm : $\widehat{V} := \arg\min_{V \in \mathbb{R}^{k \times n}} \sum_{j=1}^{n} \| S D_{W_j} U V_j - S D_{W_j} A_j \|_2^2$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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write $\widehat{V}_j = (S D_{W_j} U)^{\dagger} S D_{W_j} A_j$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

Algorithm : $\widehat{V} := \arg\min_{V \in \mathbb{R}^{k \times n}} \sum_{j=1}^{n} \|SD_{W_j}UV_j - SD_{W_j}A_j\|_2^2$

write $\widehat{V}_j = (SD_{W_j}U)^{\dagger} SD_{W_j}A_j$

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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write $\widehat{V}_j = ((SD_{W_j}U)^\top (SD_{W_j}U))^{-1} (SD_{W_j}U)^\top SD_{W_j}A_j$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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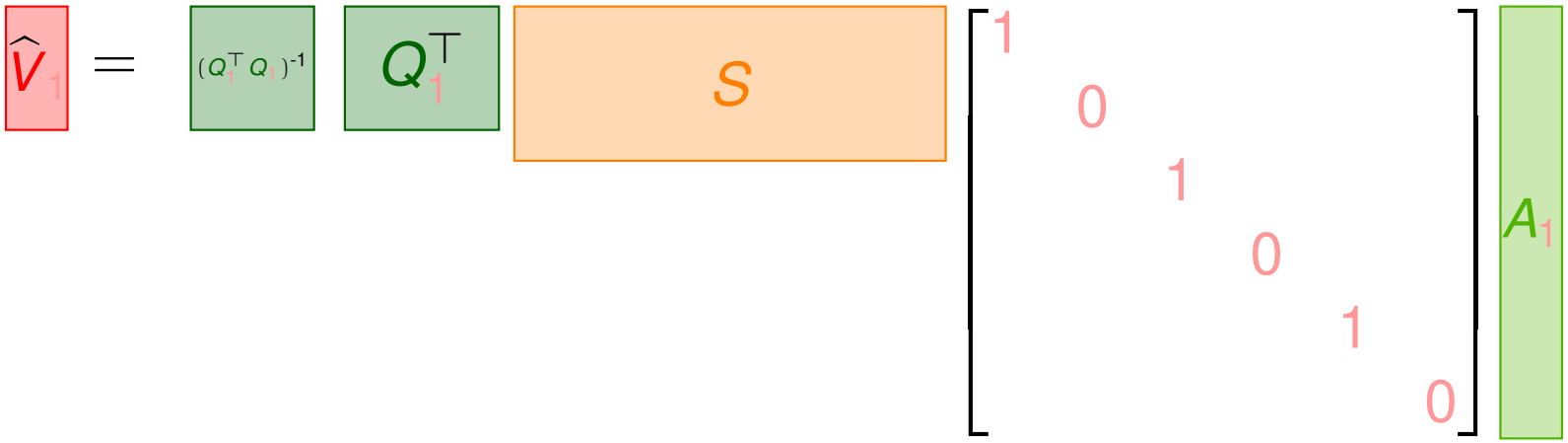
$j = 1$



Guess a Sketch

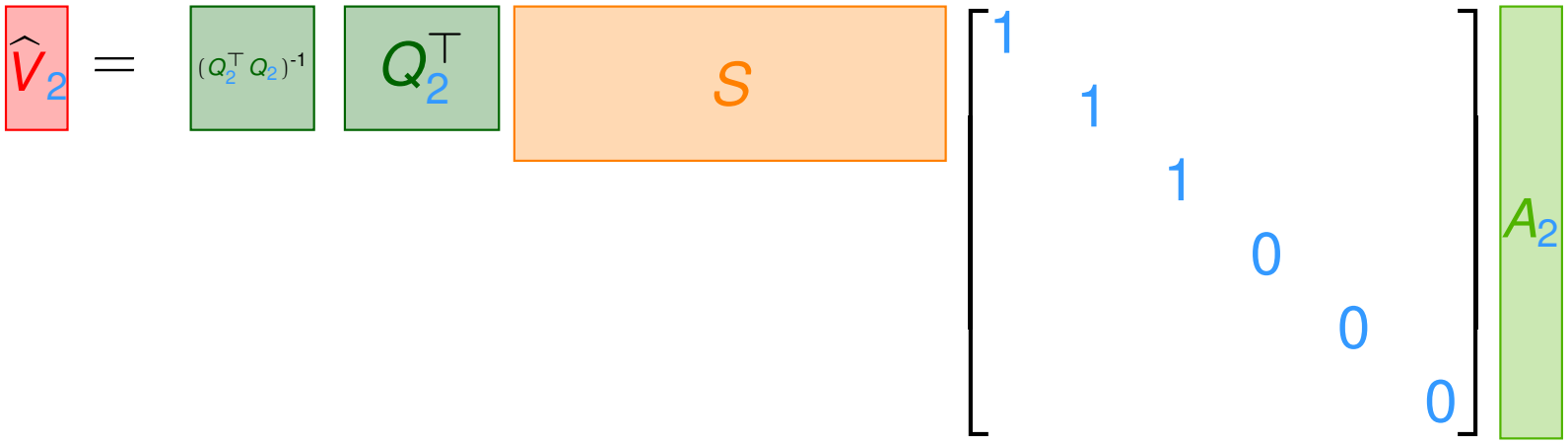
Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

Algorithm : $\widehat{V} := \arg\min_{V \in \mathbb{R}^{k \times n}} \sum_{j=1}^{n} \|SD_{W_j}UV_j - SD_{W_j}A_j\|_2^2$

write $\widehat{V}_j = (Q_j^\top Q_j)^{-1} Q_j^\top SD_{W_j}A_j$

$j = 2$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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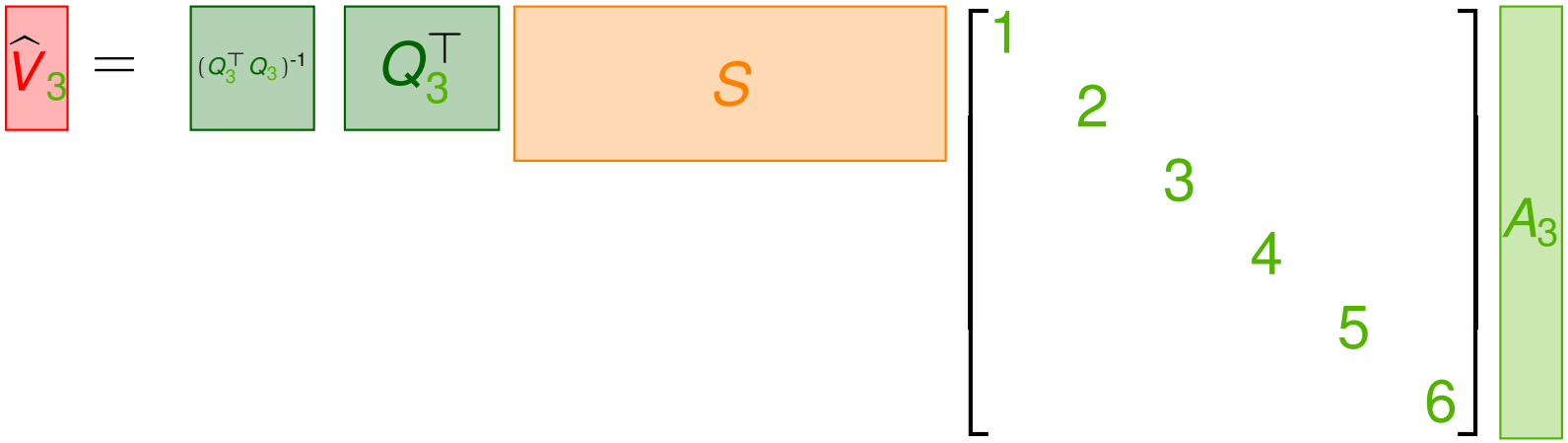
write $\widehat{V}_j = (Q_j^\top Q_j)^{-1} Q_j^\top SD_{W_j}A_j$

$j = 3$



Guess a Sketch

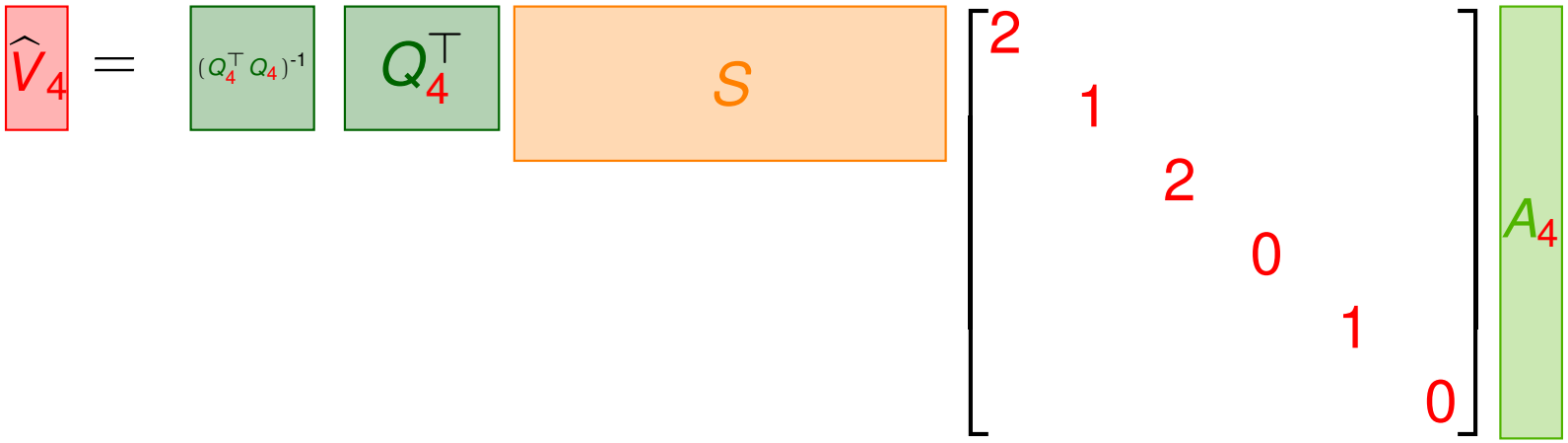
Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

Algorithm : $\widehat{V} := \arg\min_{V \in \mathbb{R}^{k \times n}} \sum_{j=1}^{n} \|SD_{W_j}UV_j - SD_{W_j}A_j\|_2^2$

write $\widehat{V}_j = (Q_j^\top Q_j)^{-1} Q_j^\top SD_{W_j}A_j$

$j = 4$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

Algorithm : $\widehat{V} := \arg\min_{V \in \mathbb{R}^{k \times n}} \sum_{j=1}^{n} \|SD_{W_j}UV_j - SD_{W_j}A_j\|_2^2$

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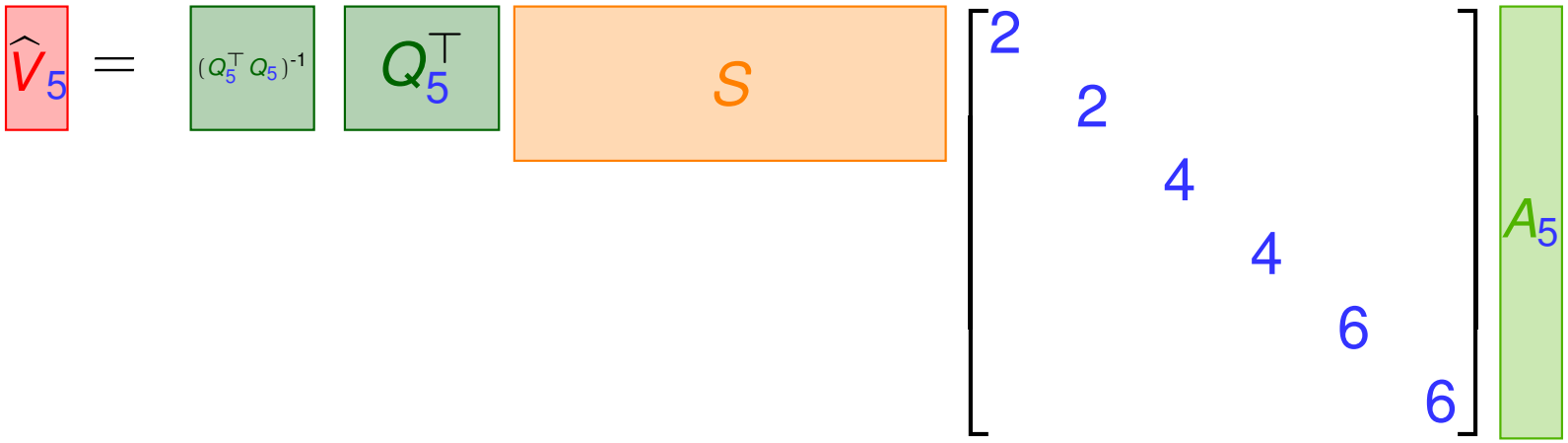
$j = 5$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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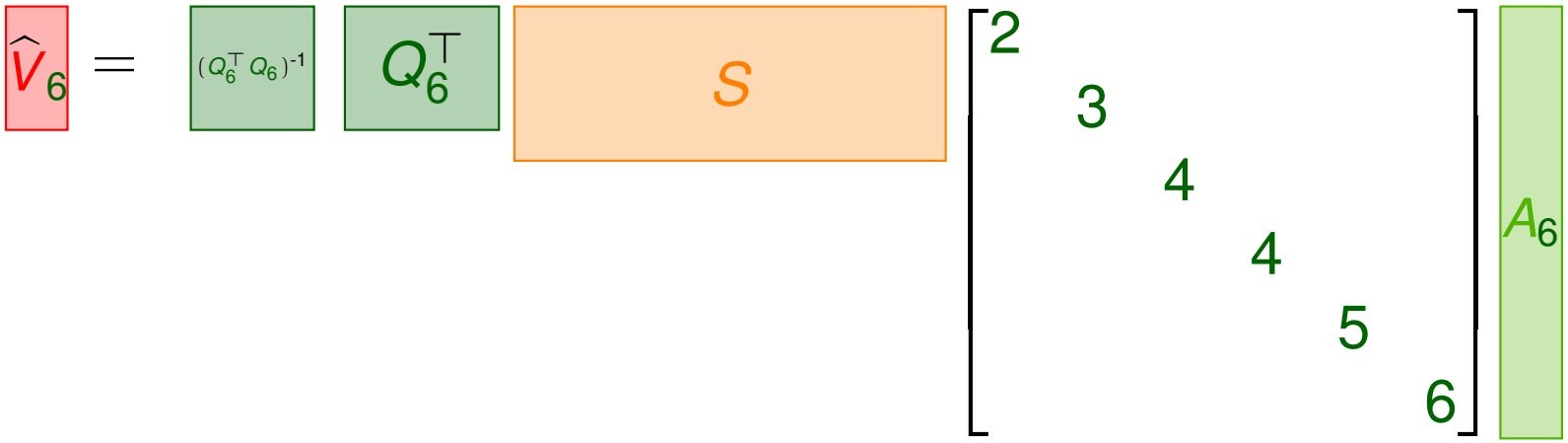
$j = 6$



Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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by Cramer's rule

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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write $\widehat{V}_j = (Q_j^\top Q_j)^{-1} Q_j^\top SD_{W_j}A_j$

by Cramer's rule

$$\boxed{(Q_j^\top Q_j)^{-1}} = \frac{1}{h_j(x)} \begin{bmatrix} f_{11}(x) & f_{12}(x) & \cdots & f_{1k}(x) \\ f_{21}(x) & f_{22}(x) & \cdots & f_{2k}(x) \\ \cdots & \cdots & \cdots & \\ f_{k1}(x) & f_{k2}(x) & \cdots & f_{kk}(x) \end{bmatrix}$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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polynomials h, f have degree $O(k)$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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Guess a Sketch

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$\forall j \in [n]$, $(Q_j^\top Q_j)^{-1}$ gives $h_j(x)$,

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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Algorithm :

$\forall j \in [n]$, $(Q_j^\top Q_j)^{-1}$ gives $h_j(x)$, then $\forall i \in [k]$, $\widehat{V}_{ij} = q_{ij}(x)/h_j(x)$

$\widehat{V}$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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$P_j = VD_{W^j}T \in \mathbb{R}^{k \times t}$ is non-degenerate

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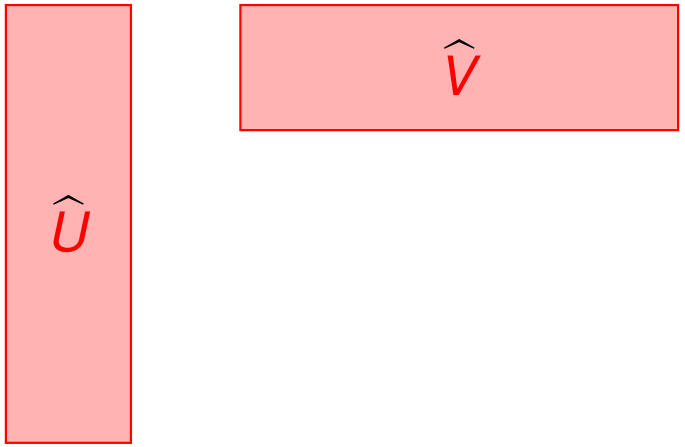

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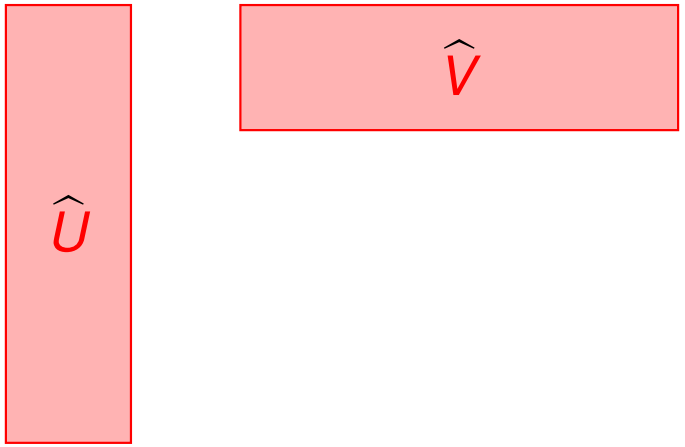
degree of $p_{ji}(x)$, $g_j(x)$, $q_{ij}(x)$, $h_j(x) = O(k)$



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degree of $p_{ji}(x)$, $g_j(x)$, $q_{ij}(x)$, $h_j(x) = O(k)$

Choose : $C \in [L^-, L^+]$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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Choose : $C \in [L^-, L^+]$

Polynomial system :

$$\|(\widehat{U}\widehat{V} - A) \circ W\|_F^2 \leqslant C$$

$$\forall j \in [n], g_j^2(x) \neq 0, h_j^2(x) \neq 0$$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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Optimize :

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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$$\forall j \in [n], g_j^2(x) \neq 0, h_j^2(x) \neq 0$$

Optimize : C by binary search over $[L^-, L^+]$

Guess a Sketch

Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

Assumption : $t = O(k/\epsilon)$, $Q_j = SD_{W_j}U \in \mathbb{R}^{t \times k}$ is non-degenerate

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degree of $p_{ji}(x)$, $g_j(x)$, $q_{ij}(x)$, $h_j(x) = O(k)$

Not done yet.

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$$\forall j \in [n], g_j^2(x) \neq 0, h_j^2(x) \neq 0$$

Optimize : C by binary search over $[L^-, L^+]$

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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We need to remove denominators.

Choose : $C \in [L^-, L^+]$

Polynomial system :

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Optimize : C by binary search over $[L^-, L^+]$

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Algorithm :

We need to remove denominators.

Denote : $G(x) = \Pi_{j=1}^{n} g_j^2(x) h_j^2(x)$ of degree $O(nk)$

Choose : $C \in [L^-, L^+]$

Polynomial system :

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Optimize : C by binary search over $[L^-, L^+]$

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Algorithm :

Denote : $G(x) = \Pi_{j=1}^{n} g_j^2(x) h_j^2(x)$ of degree $O(nk)$

Choose : $C \in [L^-, L^+]$

Polynomial system :

$$G(x) \cdot \|(\widehat{U}\widehat{V} - A) \circ W\|_F^2 \leqslant C \cdot G(x)$$

$$\forall j \in [n], g_j^2(x) \neq 0, h_j^2(x) \neq 0$$

Optimize : C by binary search over $[L^-, L^+]$

Guess a Sketch

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Algorithm :

$$(\#\,\text{constraints} \cdot \text{degree})^{O(\#\,\text{variables})}$$

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Choose : $C \in [L^-, L^+]$

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Optimize : C by binary search over $[L^-, L^+]$

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$$\left(\quad O(n) \quad \cdot \text{ degree} \right)^{O(\#\text{ variables})}$$

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Polynomial system :

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Optimize : C by binary search over $[L^-, L^+]$

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Given : $A \in \mathbb{R}^{n^2}$, $W \in \mathbb{R}^{n^2}$, $r \in \mathbb{N}$, $k \in \mathbb{N}$, $\epsilon > 0$

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$$\left(\quad O(n) \quad \cdot \quad O(nk) \right)^{O(\#\,\text{variables})}$$

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Algorithm :

$$\left(\quad O(n) \quad \cdot \; O(nk) \right)^{O(krt)}$$

Denote : $G(x) = \Pi_{j=1}^{n} g_j^2(x) h_j^2(x)$ of degree $O(nk)$

Choose : $C \in [L^-, L^+]$

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Optimize : C by binary search over $[L^-, L^+]$

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Algorithm :

$$\left(\quad O(n) \quad \cdot \; O(nk) \right)^{O(rk^2/\epsilon)}$$

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Algorithm :

$$\text{Time } (n)^{O(rk^2/\epsilon)}$$

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Optimize : C by binary search over $[L^-, L^+]$

Applications

- Guess a sketch with coding schemes

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  - Adversial matrix completion

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- Guess a sketch with [Landsberg-Manivel'04, Landsberg'12]
  - Weighted low (border) rank tensor completion
- Guess a sketch with [Arora-Ge-Kannan-Moitra'12, Moitra'13]

Applications

- Guess a sketch with coding schemes
  - Adversial matrix completion
- Guess a sketch with [Landsberg-Manivel'04, Landsberg'12]
  - Weighted low (border) rank tensor completion
- Guess a sketch with [Arora-Ge-Kannan-Moitra'12, Moitra'13]
  - Weighted nonnegative matrix factorization

Open Problems

- Can we come up a $\text{poly}(n) \cdot 2^{\text{poly}(kr/\epsilon)}$ time algorithm for WLRA problem with rank-r W or prove a $n^{\Omega(r)}$ lower bound for WLRA problem with rank-r W?

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- Can we prove a hardness result respect to parameter k or ϵ, $2^{\Omega(k)}$ or $2^{\Omega(1/\epsilon)}$ lower bound for WLRA problem?