

Algorithms, Winter 2020 at CIS

Homework 1

1. You have a neural network with three input neurons: x_1 , x_2 , and 1. Here the third neuron is always fixed to 1. Then you have a hidden layer with two ReLU activation units z_1 , z_2 , each taking as input the three inputs neurons multiplied by a weight (the weights can be different for the three inputs and the two different ReLUs). Then there is a third ReLU z which takes in the two outputs of the ReLUs z_1 and z_2 , each scaled by a weight. The output of z is the output of the network. Choose all the weights in this network so that the network implement the XOR function, namely, the output is 1 if $x_1 \neq x_2$, and the output is 0 if $x_1 = x_2$.
2. Compute the VC-dimension of the following families \mathcal{H} of functions from points in $[0, 1]$ to $\{0, 1\}$:
 - (a) \mathcal{H} is the set of all functions h that have the form $h(x) = 1_{x < t}$ for some $t \in [0, 1]$.
 - (b) \mathcal{H} is the set of all functions h that have the form $h(x) = 1_{t_1 \leq x < t_2}$ for some $0 \leq t_1 < t_2 \leq 1$.
3. Suppose B is the unit box in \mathbb{R}^2 , so it consists of (x, y) for which $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose C is the unit circle in \mathbb{R}^2 so it consists of (x, y) for which $x^2 + y^2 \leq 1$. Say which of $B \cap C$, and $B \setminus C$ are convex (there may be none or more than one).