Algorithms, February 2021 at CIS

Homework 4

1. You are given a set of r triples $(x^i, y^i, c^i) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$, as well as a number $t \geq 0$.

   (a) You would like to find an $n \times n$ matrix A such that $\langle x^i, Ay^i \rangle \geq c^i$ for all i and such that $\sum_{i=1,\ldots,n} \sum_{j=1,\ldots,n} |A_{i,j}| \leq t$ or report that no such matrix A exists. Show how to solve this problem in polynomial time.

   (b) Now you would like to find an A satisfying the requirements in part a, but among all such A, you would like to output the one for which the maximum, over i, of $\langle x^i Ay^i \rangle$, is minimized. Show how to solve this problem in polynomial time.

2. Suppose we run Seidel's algorithm on a linear program with m constraints in two dimensions and get a feasible solution x^*. Now suppose we add one more constraint $ax + by \leq c$. What is the expected time to compute a new feasible solution, or declare that no feasible solution exists? What is the worst-case time to do this?

3. Suppose you would like to find an x so as to minimize $\|Ax - b\|_\infty$ for a given $m \times n$ matrix A and $m \times 1$ vector b, where for a vector y, $\|y\|_\infty = \max_{i=1}^m |y_i|$. Show how to solve this problem with a linear program.