Algorithms, February 2021 at CIS

Homework 2

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1. Suppose I have a binary pattern P which is $n \times n \times n$ and a text T which is $m \times m \times m$, and I want to find all occurrences of P in T. Show how to solve this problem in $O(m^3)$ time, assuming operations on $O(\log(m))$ bit primes take constant time.

2. Consider the problem of finding the occurrences of any of a given set of k patterns $P_1, P_2, \ldots, P_k$ in a text T. You may assume that the size of any pattern is m, and the size of T is n. Your algorithm should use expected time $O(n + mk)$.

3. Recall in the fingerprinting lecture that we discussed the string equality problem. In this problem, Alice has a string $x \in \{0,1\}^n$ and would like to send a message M to Bob, who holds a string $y \in \{0,1\}^n$. Bob should be able to figure out if $x = y$ with probability at least $1 - \delta$.

   (a) Recall in class that we gave a few bounds on the length of the message M required to solve this problem, as a function of n and δ. Please state the best bound that we gave in class. Using $\Theta(\cdot)$ notation is fine.

   (b) If we want $\delta = 1/(10n)$, what does your bound in the previous part simplify to?

   (c) Now suppose Bob actually has n strings $y^1, y^2, \ldots, y^n$, where each $y^i \in \{0,1\}^n$, and would like to check if $x = y^i$ for each $i = 1, 2, \ldots, n$. Argue that if $\delta = 1/(10n)$, then Bob will succeed on all of the n checks simultaneously, with error probability at most $1/10$. The union bund $\mathbf{Pr}[A \text{ or } B] \leq \mathbf{Pr}[A] + \mathbf{Pr}[B]$ may be useful.