Algorithms, Winter 2020 at CIS

Homework 3

1. Suppose you have 2 items: gadgets and widgets. To make a gadget requires 30 minutes of buying material and 20 minutes to put the material together. To make a widget requires 15 minutes of buying material and 30 minutes to put the material together. The profit on a gadget is $40RMB$ and the profit on a widget is $50RMB$. The business operates for at most 8 hours per day. Write a linear program to determine how many gadgets and widgets should be made to maximize profit, and determine what the maximum profit is.

2. You are given a set of r triples $(x^i, y^i, c^i) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$, as well as a number $t \geq 0$.

   (a) You would like to find an $n \times n$ matrix A such that $\langle x^i, Ay^i \rangle \geq c^i$ for all i and such that $\sum_{i=1,\ldots,n} \sum_{j=1,\ldots,n} |A_{i,j}| \leq t$ or report that no such matrix A exists. Show how to solve this problem in polynomial time.

   (b) Now you would like to find an A satisfying the requirements in part a, but among all such A, you would like to output the one for which the maximum, over i, of $\langle x^i Ay^i \rangle$, is minimized. Show how to solve this problem in polynomial time.

3. Give an example of a linear program with d variables, $O(d)$ constraints, and 2^d vertices. Please explain your construction.