Algorithms, Winter 2020 at CIS

Homework 1

1. Suppose you are in the comparison-based model are you are given a list of n distinct numbers, $a_1, a_2, a_3, \ldots, a_n$. You are also given an integer B, and suppose B divides n. Your job is to arbitrarily partition these n numbers into B groups $G_1, \ldots, G_B$, so that

   (a) each group G_i has n/B items, and

   (b) inside of each group G_i, the numbers are sorted.

   First argue that if $B = \Theta(n)$, then this can be done deterministically using $O(n)$ comparisons. Second, show that if $B = \Theta(n^{1/3})$, then this requires $\Omega(n \log n)$ comparisons for any deterministic algorithm in the comparison-based model.

2. You have n distinct numbers that are not in sorted order, and are in the comparison model. You just want to output any subset of $n/2$ of these numbers which are in a sorted order. Show that this problem still requires $\Omega(n \log n)$ comparisons for any deterministic algorithm.

3. Suppose you have n distinct integers $\{1, 2, \ldots, n\}$ that are not in sorted order, and instead of being in the comparison-based model, you are in a model which takes in three distinct numbers a_i, a_j, and a_k and returns the median of them at unit cost. No other access to the numbers is allowed, though you can move them around for free. Show how to output either the list in sorted order or in reverse sorted order, with total cost $O(n \log n)$