

Algorithms, Winter 2020 at CIS

Homework 1

1. Suppose you are in the comparison-based model and you are given a list of n distinct numbers, $a_1, a_2, a_3, \dots, a_n$. Define the *rank* of an item a_j to be its position i , in a sorted order of these numbers. So if π is a permutation from $\{1, 2, \dots, n\}$ to $\{1, 2, \dots, n\}$, and $a_{\pi(1)} < a_{\pi(2)} < \dots < a_{\pi(n)}$, then the rank of a_j is equal to the index i for which $\pi(i) = j$.

Give a deterministic algorithm for outputting the entire set of items of rank 3^i , for $i = 0, 1, 2, 3, 4, \dots, \lfloor \log_3 n \rfloor$. Your algorithm should use $O(n)$ comparisons.

2. True or False: given a list a_1, \dots, a_n , one can output a sorted list of the smallest $n^{1/3}$ items in $O(n)$ time.
3. What is an algorithm with the smallest number of comparisons you can find for outputting both the maximum and the minimum of n numbers?