Algorithms, 2021 at CIS

Homework 1

1. Suppose we have n experts and T days, and on each day each expert predicts a value in $\{0, 1\}$. Suppose our strategy is the following: on the i-th day, we choose the prediction made by an expert that has made the fewest mistakes so far, breaking ties arbitrarily. Give an example where we could be incorrect on every day, but the best expert is only incorrect on a $1/T$ fraction of days.

2. Suppose we are making binary predictions again, and instead of having a single expert that makes no mistakes, we have s experts that all make no mistakes, for some values of s, $1 < s \leq n$. Give a prediction strategy and prove that it makes at most $\lceil \log_2 n/s \rceil$ mistakes.

3. In the randomized weighted majority algorithm, suppose for the best expert i we start with a weight $w_i = (n + 1)/2$, while for every other expert we start with a weight of $1/2$. What should we set ϵ to be to minimize our error rate in the analysis now?