Algorithms, Winter 2020 at CIS

Homework 3 **Due: 2/9/20 11:59pm China time**

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1. Let $U = 2^u$ and $M = 2^m$. Prove that the family of hash functions $A \cdot x + b \bmod 2$, where $A \in \{0,1\}^{m \times u}$ is a random binary matrix, and $b \in \{0,1\}^m$ is a random binary vector, is a 2-universal family.

2. Suppose Alice has a bit string $x \in \{0,1\}^n$ and Bob has a bit string $y \in \{0,1\}^n$. They are promised that there is exactly one index $i \in \{1,2,3,\ldots,n\}$ for which $x_i \neq y_i$, and for all $j \neq i$, $x_j = y_j$. Alice sends a single message to Bob, and Bob needs to figure out what the value of i is with probability at least 4/5. Show that there is a way to do this where Alice's message length is only $O(\log n)$ bits long.

3. Consider the following 2-player zero sum game:

$$\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

   Calculate the minimax optimal strategies of the row player and the column player, as well as the value of the game.