Algorithms, May-June 2020 at CIS

Homework 1

1. Suppose you have two length-n binary integers a and b and n is a power of 3, and you would like to multiply a and b quickly. Suppose, in $O(n)$ time, you could create 5 length-$n/3$ numbers $p_1, \ldots, p_5$ and 5 length-$n/3$ numbers $q_1, \ldots, q_5$, so that after computing $p_1 \cdot q_1, p_2 \cdot q_2, \ldots, p_5 \cdot q_5$ you could combine the results, in $O(n)$ time, to compute $a \cdot b$. What is the overall big-Oh running time of your algorithm to multiply a and b?

2. Suppose you could multiply two arbitrary 4×4 matrices A and B using 31 multiplications and 71 additions. Using this to devise a divide and conquer algorithm, along the lines of Strassen's, what would your running time be for multiplying two $n \times n$ matrices? Big-Oh notation is fine.

3. Suppose you have a degree-2 polynomial $p(x)$ for which $p(0) = 5$, $p(1) = 2$, and $p(2) = 3$. Then $p(x) = ax^2 + bx + c$. Solve for a, b, and c.