# Algorithms, Summer 2019 at CIS

**Homework 3** **Due: 7/26/19 before class**

1. You are given a set of $r$ triples $(x^i, y^i, c^i) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$, as well as a number $t \geq 0$.
   (a) You would like to find an $n \times n$ matrix $A$ such that $\langle x^i, Ay^i \rangle \geq c^i$ for all $i$ and such that $\sum_{i=1,\ldots,n} \sum_{j=1,\ldots,n} |A_{i,j}| \leq t$ or report that no such matrix $A$ exists. Show how to solve this problem in polynomial time.
   (b) Now you would like to find an $A$ satisfying the requirements in part a, but among all such $A$, you would like to output the one for which the maximum, over $i$, of $\langle x^i A y^i \rangle$, is minimized. Show how to solve this problem in polynomial time.
2. Give an example of a linear program with $d$ variables, $O(d)$ constraints, and $2^d$ vertices.
3. Suppose you are given an unweighted, undirected graph $G$ on $n$ vertices. Recall in class we showed the existence of a 2-additive spanner $H$ with $O(n^{3/2})$ edges. The graph $H$ was a subgraph of $G$ and had the property that for every pair of vertices $u$ and $v$ in $G$, $d_G(u,v) \leq d_H(u,v) \leq d_G(u,v) + 2$.

   Now suppose we do not require that $H$ be a subgraph, and further, we allow the edges of $H$ to be weighted. Show how, with probability $1 - 1/n$, to find such an $H$ with $O(n^{4/3} \log^2 n)$ edges such that for every pair of vertices $u$ and $v$ in $G$, $d_G(u,v) \leq d_H(u,v) \leq d_G(u,v) + 4$. Note that the distance in $H$ is a weighted distance, that is, it is the sum of weights along the path from $u$ to $v$ in $H$ with the minimum possible sum of weights.

   Hint: recall that in our 2-additive spanner construction, we separated edges based on those which had one endpoint of degree at most $n^{1/2}$, and those with both endpoints of degree larger than $n^{1/2}$. Here, consider separating edges based on those which have one endpoint of degree at most $n^{1/3}$, and those with both endpoints of degree larger than $n^{1/3}$. You will need to change the algorithm in the case that both enedpoints have degree larger than $n^{1/3}$. Think about sampling a random set of $O(n^{2/3} \log n)$ vertices and arguing similarly to the 2-additive construction, but use the fact that $H$ is allowed to be weighted.