

# Algorithms, Summer 2019 at CIS

## Homework 1

Due: 7/18/19 before class

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1. Give an example of a pair of functions  $f(n)$  and  $g(n)$  for which  $f(n) \neq \Omega(g(n))$  and  $f(n) \neq O(g(n))$ .
2. Suppose I have a number  $i \in \{1, 2, 3, \dots, n\}$  and you are allowed to ask me questions of the form “is  $i \leq t$ ” for a number  $t \in \{1, 2, \dots, n\}$ . You can ask multiple questions for different values of  $t$ , as we did in class, let’s call these  $t_1, t_2, \dots$ . We saw in class that if there is at most one value  $t_j$  asked with  $t_j > i$ , then it is possible to ask  $O(\sqrt{n})$  queries to figure out the value of  $i$ . What if we instead allow for at most 100 values  $t_j$  asked to have the property that  $t_j > i$ ? How many queries are enough to figure out the value of  $i$  now? You can use big-Oh notation in your answer.
3. The variance  $\mathbf{Var}[X]$  of a random variable  $X$  is defined to be  $\mathbf{E}[|X - \mathbf{E}[X]|^2]$ . Prove that  $\mathbf{Var}[X] = \mathbf{E}[X^2] - \mathbf{E}^2[X]$ .