1. Make a Piazza comment related to Chapter 2 if you have not done so yet.

2. Make a Piazza comment related to Chapter 3 if you have not done so yet.

3. Suppose matrix $A$ is a database of restaurant ratings: each row is a person, each column is a restaurant, and $a_{ij}$ represents how much person $i$ likes restaurant $j$. What might $v_1$ represent? What about $u_1$? How about the gap $\sigma_1 - \sigma_2$? (There are multiple reasonable answers here).

For this specific question, feel free to combine with question 2 and either post your thoughts or comment on someone else’s thoughts, or post thoughts about what $v_2, u_2$ might represent, etc. You could also read up and make a post about how one might do SVD when $A$ has missing entries (not all people have ranked all the restaurants).

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \\ 1 & -2 \\ -1 & -2 \end{bmatrix}$$

(a) Run the power method starting from $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $k = 3$ steps. What does this give as an estimate of $v_1$?

(b) What actually are the $v_i$’s, $\sigma_i$’s, and $u_i$’s? It may be easiest to do this by computing the eigenvectors of $B = A^T A$.

5. Prove that $u_1$ is the first singular vector for $A^T$ (let’s assume $\sigma_1 > \sigma_2$ so we can say “the” instead of “a”). Hint: we know the power method applied to $C = AA^T$ will approach the first singular vector of $A^T$ if it approaches any fixed vector at all.

6. Consider a set $S$ of $n$ points in $\mathbb{R}^d$ whose center of mass is the origin. As we have been discussing in class, the first singular vector (for a matrix in which each of these points is a row) gives the line through the origin that minimizes the sum of squared distances of all the points in $S$ to that line. What if we allowed lines not through the origin? Here we will prove that such lines don’t help. Specifically, for any line $\ell$, the line $\ell'$ parallel to $\ell$ that goes through the origin is at least as good.

Formally, fix some line $\ell$. Let $b$ be the point on $\ell$ closest to the origin and let $v$ be a unit length vector parallel to $\ell$. Mathematically, we have $\ell = \{b + \lambda v : \lambda \in \mathbb{R}\}$ (convince yourself of this fact before going on). Let $\ell' = \{\lambda v : \lambda \in \mathbb{R}\}$ be the line parallel to $\ell$ that goes through the origin.
(a) Explain why for any point \( x \), its squared distance to \( \ell' \) is \( ||x||^2 - (v \cdot x)^2 \).

(b) What is the analogous formula for distance squared of a point \( x \) to \( \ell \)?

(c) Now prove the theorem: prove that the sum of squared distances of points in \( S \) to \( \ell' \) is less than or equal to the sum of squared distances of points in \( S \) to \( \ell \). Be clear where you use the fact that the center of mass of \( S \) is the origin.