Geometry of binocular imaging III : Wide-Angle and Fish-Eye Lenses

Victor S. Grinberg
M. W. Siegel

Robotics Institute, School of Computer Science, Carnegie Mellon University
5000 Forbes Ave., Pittsburgh, PA, 15213

ABSTRACT

We address the issue of creating stereo imagery on a screen that, when viewed by naked human eyes, will be indistinguishable from the original scene as viewed through a visual accessory. In doing so we investigate effects that appear because real optical systems are not ideal. Namely, we consider optical systems that are not free from geometric aberrations. We present an analysis and confirming computational experiments of the simulations of stereoscopic optical accessories in the presence of aberrations. We describe an accessory in the framework of the Seidel-Schwarzschild theory. That means that we represent its deviation from an ideal (Gaussian) device by means of five constants. Correspondingly, we are able to simulate five fundamental types of *monochromatic* geometric aberrations: spherical aberration, coma, astigmatism, curvature-of-field, and distortion (barrel and pincushion). We derive and illustrate how these aberrations in stereoscopic optical systems, can lead to anomalous perception of depth, e. g., the misperception of planar surfaces as curved, or even twisted, as well as to circumstances under which stereoscopic perception is destroyed. The analysis and numerical simulations also allow us to simulate the related but not identical effects that occur when lenses with aberrations are used in stereoscopic cameras.

Keywords: autostereoscopic, binocular imaging, stereo imaging, binoculars, stereomicroscope, periscope, geometric aberration

1. INTRODUCTION

Binocular (stereoscopic) imaging is one of the most powerful means to use 2D images for realistically simulating the appearance of 3D objects when viewed with two eyes. All binocular imaging systems must provide a means for presenting one image to the left eye and another to the right. We investigate the geometric issues related to binocular imaging. We consider a system where images for both the left and right eyes are created by appropriate software on one computer screen, and where a means to ensure that each eye sees only the intended image is employed. Our goal is to study the "correct geometry" of this process, i.e., how synthetic objects existing in the synthetic 3D world must be mapped onto the screen for the truest binocular perception. We concentrate on these pure geometric issues, putting aside other effects that can contribute to depth perception, such as color, lighting, texture, etc.

In the first[3] of two previous papers we concentrated on the naked-eye case. We analysed the geometry that must be precisely implemented to satisfy the requirement that the imagery be geometrically indistinguishable from the reality that would be perceived by the "naked" human visual apparatus. The results applied identically to both camera-captured and computer-generated images: it taught in the first case how to build a stereo-camera, and in the second case how to give the computer a camera model.

Then in the second paper[4] we considered the augmented-eye case. Namely we investigated the geometry needed to create imagery on a screen that, when viewed by naked human eyes, would be indistinguishable from the original scene as viewed through a visual accessory. We use the generic term *optical device* for such an accessory. Typical stereoscopic optical devices of interest are binoculars, stereomicroscopes, and binocular periscopes. The considered devices were assumed free of aberrations, i.e., they were assumed to obey the laws of "Gaussian" optical systems. This result also applied identically to both camera-captured and computer-generated images: we could equally easily imagine either looking directly through the optical device, or replacing the eyes with cameras whose images were later viewed stereoscopically.

In this paper we consider the fact that real optical devices are often rather far from the Gaussian case. This deviation is responsible for several types of geometric aberrations. In stereoscopic optical devices, combinations of aberrations in the left and right optical systems can cause a variety of potentially undesirable effects. This paper is devoted to theoretical investigation and computer simulation of effects that are caused by geometric aberrations of stereo systems. Among the five classical aberrations, the most important in this context is *distortion*. It is frequently present, e. g., in wide-angle and "fish-eye" lenses, where it is an inevitable consequence of obtaining the desired large field of view. The consequences to stereoscopy are important, since the need for wide viewing angle and 3D-perception go together in many applications. Thus being able to model the stereoscopic perceptual effect of aberrations permits us to predict the consequences of having these aberrations in real optical systems without having first to build them and test them on human subjects. In this case there are some differences between direct vision systems and systems with an intervening planar image capture stage.

Although it is not the main topic of this paper, we also point out here that there are important consequences in the more general field of computer generation of photo-realistic imagery: all modern computer graphics systems have the built-in capability to rather easy simulate Gaussian optics, but not real optical systems with aberrations. This is so because all computer graphics is based on the projective transformation, which coincidentally describes Gaussian (but not non-Gaussian) optical systems[4].

2. FORMAL DESCRIPTION OF GEOMETRIC ABERRATIONS

In this section we give the mathematical description of geometric aberration and introduce notation that will be used later. The material is not new (see, for example[1] and[2]), but we present it here in the form that we need.

The aberrations of an optical device characterize its deviation from an ideal device. The ideal optical device is by definition a combination of ideal optical elements such as mirrors and lenses. In an ideal device all rays originating at a point, after passing through the device, and after suitable extension of the exit rays, intersect at another point, the *image* (real or virtual) of the original one. For an ideal optical element the positions of the point and its image are linked by the formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad ,$$

where d_o is the "object distance", d_i is the "image distance", and f is the "focal length". Devices that satisfy this condition are called *Gaussian*. As mentioned in passing above, this fundamental relationship (and its consequences with respect to the relative sizes of object and image, i.e., magnification) can be expressed as a projective transformation[4].

In the presense of aberration the course of rays deviates from the Gaussian prescription. As a consequence the rays originating at a point do not generally intersect at any point. The course of rays in the absence and presense of aberration is sketched in Fig. 1.

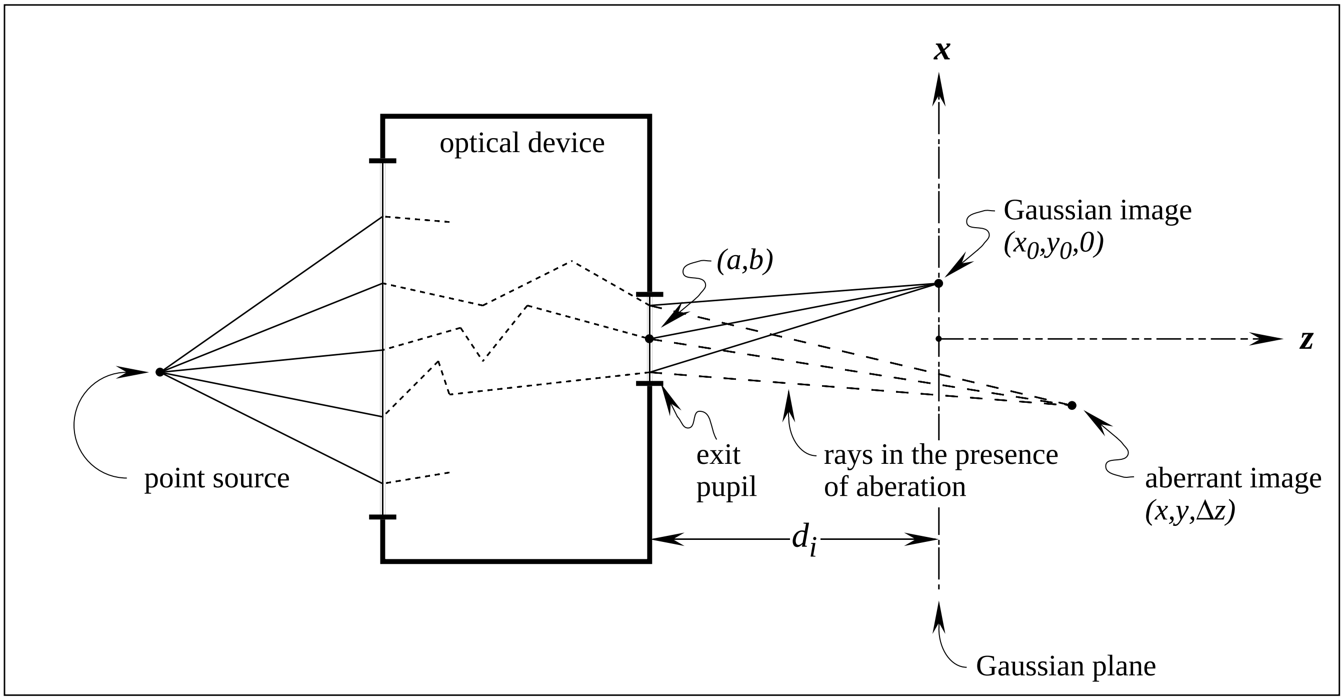

Figure 1

To describe aberrations mathematically one needs to introduce appropriate parametrization of all rays and define the deviation of a ray from its Gaussian course in terms of these parameters. Four parameters are needed to specify a ray. Let us consider the plane that is orthogonal to the optical axis and contains the exit pupil (*exit pupil plane*), and introduce in it an orthonormal system of coordinates centered at the optical axis. We denote by (a, b) the coordinates of a point where a ray intersects this plane. Then we consider the plane that is orthogonal to the optical axis and contains the Gaussian image of the point (*Gaussian image plane*). We introduce an orthonormal system of coordinates in this plane such that its axes are parallel to the axes that we previously created in the exit pupil plane. The coordinates of a point where an ideal ray intersects the Gaussian image plane (that is the Gaussian image) will be denoted by (x_0, y_0). The quadruple (a, b, x_0, y_0) will then serve as a parametrization of a ray.

Because of aberration, real rays do not intersect the Gaussian image plane at the location of the Gaussian image. The coordinates of intersection of an actual ray with the plane will be denoted by (x, y). The deviation caused by aberration is thus described by the pair $\Delta x = x - x_0$ and $\Delta y = y - y_0$. The mathematical description of aberration must express Δx and Δy as functions of a, b, x_0, y_0.

In the framework of the Seidel-Schwarzschild theory these expressions are[1,2]

$$\Delta x = x_0 (2 C \kappa^2 / d_i - E r^2 / d_i^2 - F \rho^2) + a (B \rho^2 d_i + D r^2 / d_i - 2 F \kappa^2) \ ,$$

$$\Delta y = y_0 (2 C \kappa^2 / d_i - E r^2 / d_i^2 - F \rho^2) + b (B \rho^2 d_i + D r^2 / d_i - 2 F \kappa^2) \ ,$$

where $r^2 = x_0^2 + y_0^2$, $\rho^2 = a^2 + b^2$, $\kappa^2 = a x_0 + b y_0$, and d_i is the distance between the exit pupil plane (for a simple lens without stops, this is the lens position) and the Gaussian image plane (see Fig. 1).

Since a, b, x_0, y_0 are all of the same order-of-magnitude as r, we can see that these expressions for Δx and Δy contain only terms the order of r^3. Thus the Seidel-Schwarzschild theory is called "third-order", in contrast to the Gaussian theory, which is "first-order". Coefficients B, C, D, E, F are characteristic of the optical device, and describe its aberration properties. Each of these coefficients corresponds to a particular aberration, namely coefficient B is responsible for *spherical aberration*, coefficient C is responsible for *astigmatism*, coefficient D is responsible for *curvature-of-field*, coefficient E is responsible for *distortion*, and coefficient F is responsible for *coma*.

3. HOW ABERRATION AFFECTS STEREO PERCEPTION

Aberration has two effects: 1) it makes images of points fuzzy and 2) it shifts the images from their Gaussian position. We are especially interested in the situation when the aberration has *only* the second effect, i.e., in the case when aberrant rays originating at a point source converge at some point displaced from the Gaussian image of the source. It can be shown (see[1], pp. 217-218) that this is the case if and only if an optical device is free of spherical aberration, astigmatism and coma, i.e., when $B=C=F=0$. Because we are primarily interested in the perceptual effects of shape changing but not in sharpness changing aberrations, and because as a practical matter good lenses are sharp (perhaps at the expense of shape presentation), in the remainder of this paper we restrict our considerations to *distortion* and *curvature-of-field*. Similarly, whenever we remark on the consequences of these considerations to planar image capture, we ignore depth-of-focus effects.

As long as one is interested only in deviation of the point image *in* the Gaussian plane (*in-plane* deviation), any accompanying deviation *from* the Gaussian plane (*out-of-plane* deviation) does not matter. This situation prevails, for example, in photography, where the image space is sampled by a planar film or sensor. But in studying stereo perception of *volumetric* images affected by aberration we must consider both deviations. The preservation or obscuration of out-of-plane effects is what leads to the difference between directly viewed and photographically or electronically captured image pairs.

To describe quantitatively the out-plane deviation of the point image we introduce a third co-ordinate, *z*, orthogonal to the Gaussian plane. Correspondingly we denote this deviation by Δz. Consider the point (x,y) where a ray intersects the Gaussian plane as a function of the point (a,b) where the ray intersects the exit pupil plane. Assuming that all rays intersect in a point (with the third coordinate Δz), i.e., $B=C=F=0$, one can easily obtain from Fig. 1 that $dx/da = \Delta z/(d_i + \Delta z) \approx \Delta z/d_i$. On the other hand the Seidel-Schwarzschild formulas gives us in this case $dx/da = Dr^2/d_i$. Combining these two expressions we have

$$\Delta z = Dr^2 .$$

Notice that:

(1) *distortion* (E) does not appear in this expression: *distortion* moves image points around in the Gaussian plane, but not out-of-plane;

(2) *curvature-of-field* (D) shifts image points out-of-plane;

(3) whereas the in-plane effects of both D and E are third order, the out-of-plane effect of D is second-order: the out-of-plane effect, i.e., the front-back distortion of an image suffering from *curvature-of-field*, is large relative to the in-plane effects.

In studying the stereo perception of images affected by aberration we must pay attention to two effects. The first effect is an obvious one: out-plane deviation becomes "visible" as a front-back distortion due to the depth perception that is guaranteed by direct stereo vision. The second effect is less obvious: the in-plane deviation may change the disparity between left and right images and therefore incorrectly "place" the image visibly away from the Gaussian plane. Thus even in the case of pure distortion, when stereo images lie in the Gaussian plane, a human observer may perceive the scene shifted front-back and twisted due to the dependence of front-back shift on left-right and up-down positions. The latter effects are visible in both direct viewed systems and those with intervening cameras.

4. SIMULATING EXPERIMENTS

We designed and created a software system for simulating the phenomena predicted above. For most of the experiments described in this paper we used an SGI workstation equipped with Toshiba LCD shutter goggles toggled via synchronous signals sent to the workstation's parallel port. We used this system in preference to the StereoGraphics Crystal Eyes wireless goggles we normally use for viewing 3D-computer graphics and 3D-digital video because in order to obtain real-time performance we need to work in small window using a double buffering technique that is incompatible with the StereoGraphics paradigm. This environment allows us to sequentially create the left and right projection of a scene on the screen and send each image to the viewer's corresponding eye. The effect of the optical device through which the scene is viewed is simulated by corresponding calculations that include aberration effects. The system includes a variety of parameters that can be controlled interactively: the position and orientation of the viewed objects, the position and orientation of the left and right optical devices, and the parameters of these devices, such as the focal length and the aberration constants D, E. Fig. 2 shows the flow chart of simulations.

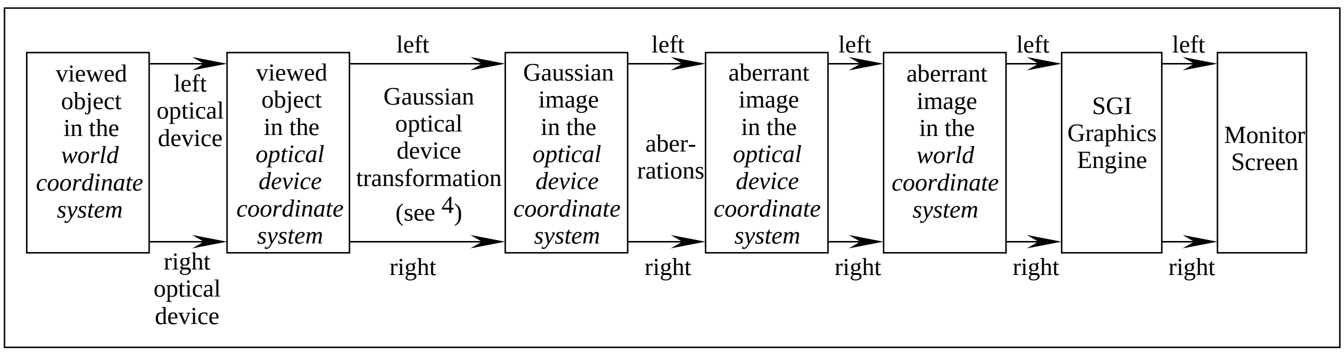

Figure 2: Flow chart of the Simulation

The simulation showed a principal difference between two cases: when both eyes use the same optical device (as in periscopes), and when each eye uses its own optical device (as in binoculars or stereomicroscopes). In the second cases we assumed that the two devices are identical and placed symmetrically relative to the bridge of the nose.

If both eyes use the same optical device we deal with just one image that is viewed stereoscopically; the simulation just lets us see the in-plane and out-of-plane shifts caused by aberration. Each object point has *one* corresponding image point; the three dimensional aerial image may be distorted in-plane and out-of-plane, but it can be viewed without difficulty. Fig. 3-6 show examples of simulation with one optical device.

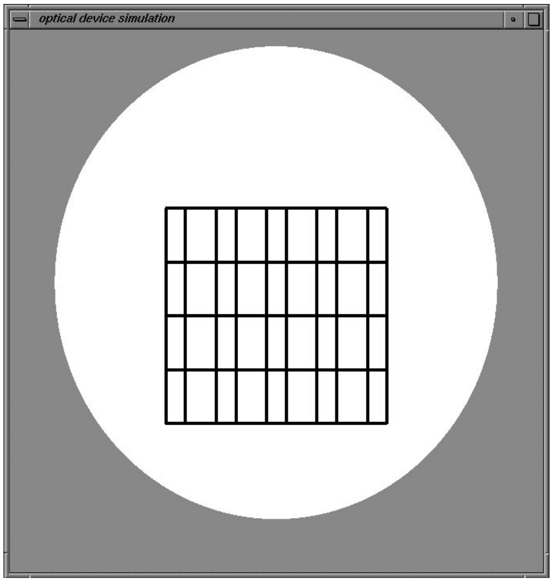

Figure 3: 1 lens, regular grid, no aberration;
images: two 4×4 grids displaced left and right of center

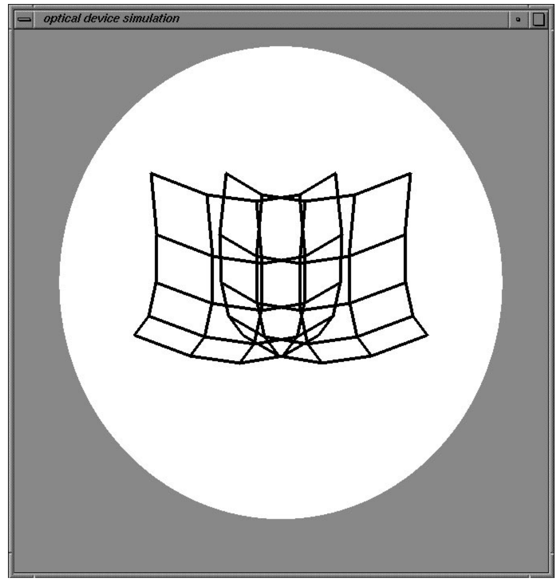

Figure 4: 1 lens, pure curvature-of-field ($D \neq 0$)
no vertical disparities;
fuses into a curved surface as expected

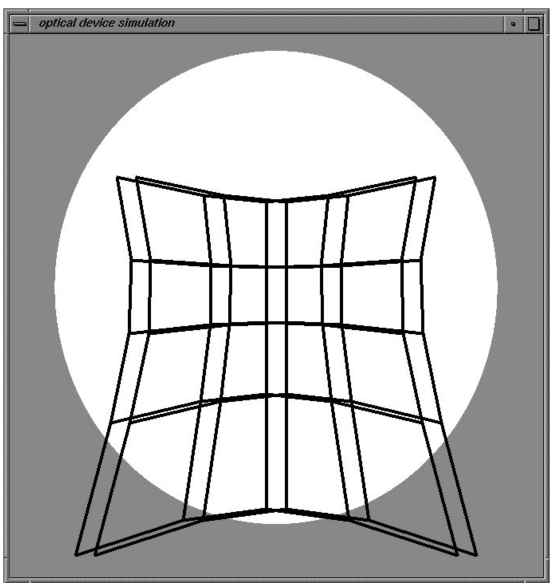

Figure 5: 1 lens, pincushion distortion ($E < 0$)
no vertical disparity;
fuses into a distorted but planar figure

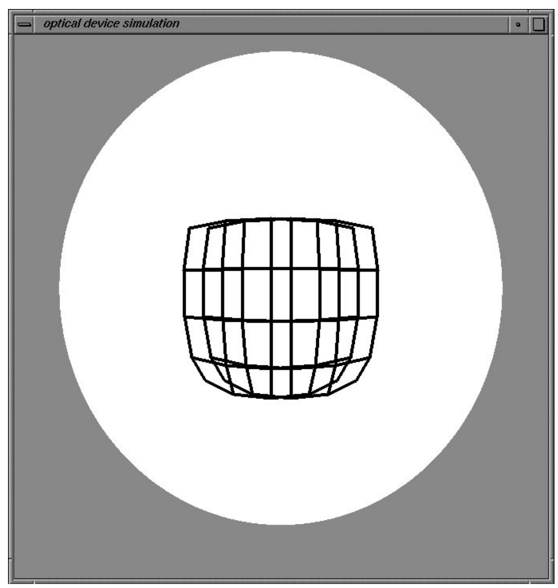

Figure 6: 1 lens, barrel distortion ($E > 0$)
no vertical disparity;
fuses into a distorted but planar figure

The situation dramatically changes when each eye uses its own optical device. In this case we deal with *two* aerial images created by two devices. In the absence of aberration these two images project on the screens or sensors with corresponding points having only horizontal displacement and disparity, so the eyes can fuse them, re-creating a single 3D image in space. However in the presense of aberration these aerial images and their projections on screens or sensors typically have vertical disparity and therefore cannot be fused. These effects are shown in Fig. 7-10.

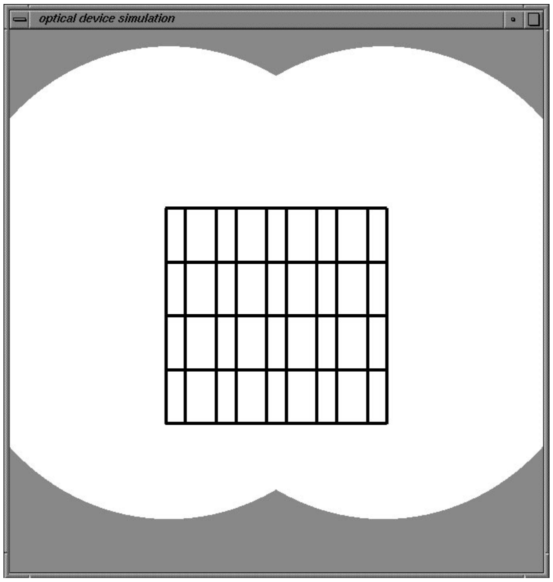

Figure 7: 2 lenses, regular grid, no aberration;
images: two 4×4 grids displaced left and right of center

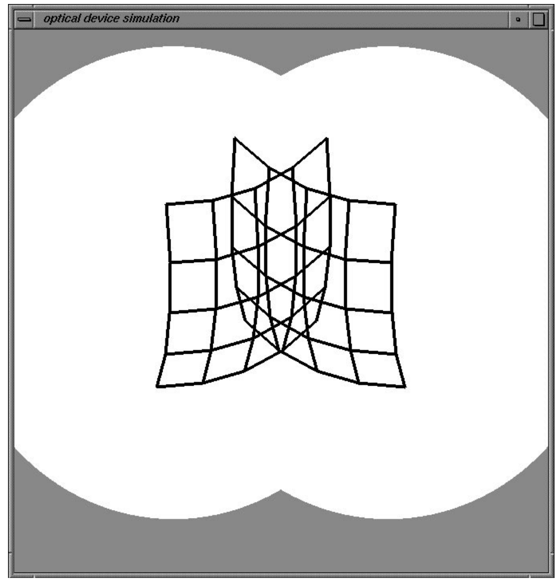

Figure 8: 2 lenses, pure curvature-of-field ($D \neq 0$)
vertical disparity;
cannot be fused

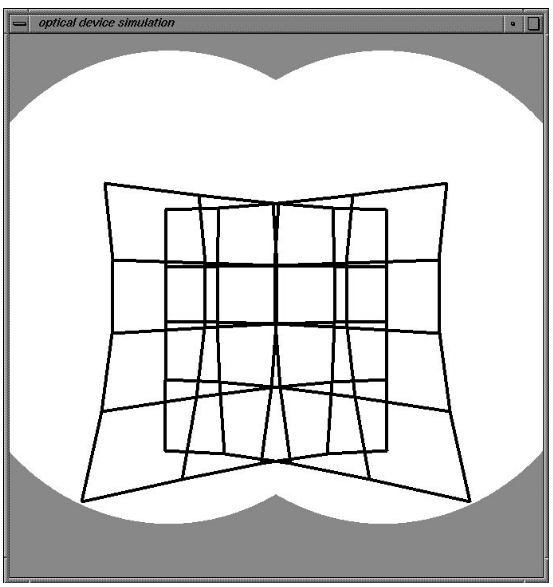

Figure 9: 2 lenses, pincushion distortion ($E < 0$)
vertical disparity;
cannot be fused

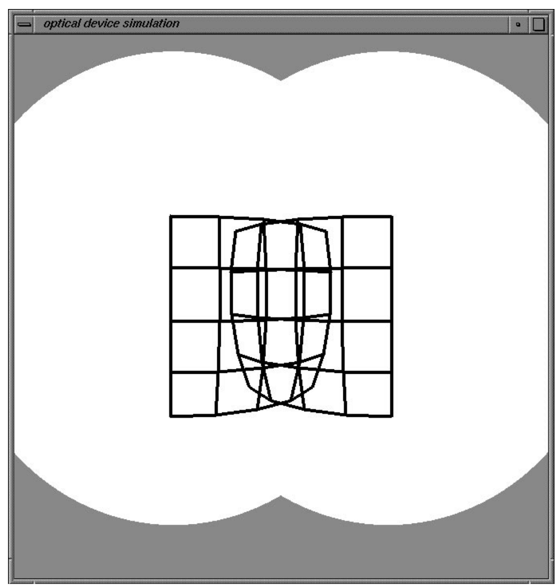

Figure 10: 2 lenses, barrel distortion ($E > 0$)
vertical disparity;
cannot be fused

Despite the fact that the two-device case results in images that cannot be fused, we can nevertheless visualize the effect if we create a special scene: a set of vertical straight lines. For this scene vertical disparity cannot be perceived, so the viewer can fuse the two images even in the presence of aberration. Depending on the relative position and orientation of the viewer, the scene, and the optical device a variety of anomalous "depth" displacements are observed. These displacements are different for different parts of the object; thus flat surfaces, even those parallel to the Gaussian plane, look twisted. The effects are shown in Fig. 11-13.

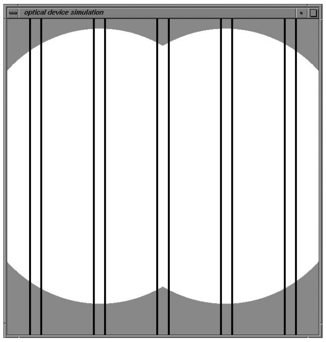

Figure 11: 2 lenses, vertical lines, no aberration; images: two sets of vertical lines displaced left and right of center

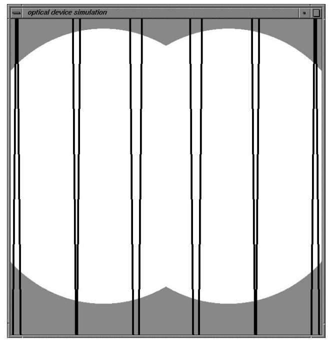

Figure 12: 2 lenses, vertical lines, pure curvature-of-field ($D \neq 0$), vertical disparity is invisible; fuses into a curved surface

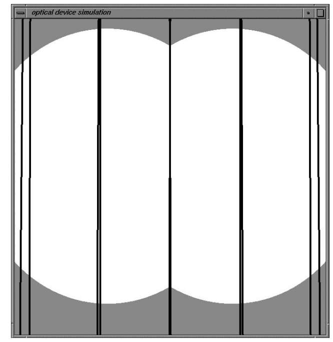

Figure 13: 2 lenses, vertical lines, barrel distortion ($E < 0$), vertical disparity is invisible; fuses into a curved surface

Fig. 4, 5, 6, 8, 9, 10, 12, 13 were obtained using intentionally exaggerated values of D and E: $D \sim 0.003 - 0.007\ mm^{-1}$, $E \sim 0.1 - 0.2$.

5. CONCLUSION

We reviewed naked-eye and augmented-eye stereoscopic systems using Gaussian optics, and introduced the issue of aberrations in optical systems. We briefly reviewed the Seidel-Schwarzschild framework of geometrical aberrations; *distortion* (E) and *curvature-of-field* (D) are of particular interest because they preserve image sharpness but differentially displace image point locations, thus potentially adversely affecting stereo perception. We noted the important distinction between single-axis stereoscopic systems and dual-axis stereoscopic systems: in the former case distortion and curvature-of-field change the appearance of the scene, but not its viewability; in the latter case, the differential aberration effects between the two aerial images create vertical disparities that can make it impossible to fuse them.

We have written a computer program to model and render these effects stereoscopically on a computer screen; the results are illustrated in the accompanying figures. We believe this program will be useful for pre-viewing the performance, in contemplated stereoscopic systems, of lenses whose aberration coefficients are known. We believe it will also be useful in human factors or psychophysical experiments to characteristize the relative importance of various aberrations, and combinations of aberrations, on human stereoscopic perception.

6. ACKNOWLEDGMENTS

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7. REFERENCES

1. Max Born and Emil Wolf, Principles of Optics, Pergamon Press, Oxford, New York, 1969.
2. W. T. Welford, Aberrations of optical systems, Bristol, Boston, 1986.
3. V. S. Grinberg, G. W. Podnar, M. W. Siegel, "Geometry of Binocular Imaging", in *Stereoscopic Displays and Applications V*, Proceedings of the 1994 IS&T/SPIE Conference on Electronic Imaging: Science & Technology, San Jose, California, USA, SPIE, 8-10 Feb., 1994, pp. 56-65.
4. V. S. Grinberg, G. W. Podnar, M. W. Siegel, "Geometry of binocular imaging II : The augmented eye", in *Stereoscopic Displays and Applications VI*, Proceedings of the 1995 IS&T/SPIE Conference on Electronic Imaging: Science & Technology, San Jose, California, USA, SPIE, 7-8 Feb., 1995, pp. 142-149.