Algorithms in the Real World (15-853), Spring 2018
Assignment #6 Due: Tuesday, April 24, 2018

Complete all problems.

You are not permitted to look at solutions of previous years' assignments. You can work together in groups, but all solutions must be written up individually, and please list your collaborators. If you get information from sources other than the course notes and slides, please cite the information, even if from Wikipedia or a textbook.

For algorithm design problems, you should try to provide pseudocodes or pseudocode-like well organized verbal description. Then you should analyze the complexity (work and span) of your algorithms based on the pseudocodes. Please make your solutions as clear as possible, and they will be graded by the correctness and clearness. If only some "high-level" concepts are provided, you will get no more than half of the credits even if they are correct.

Problem 1: Chernoff Bound Redux [15 pts]

Recall the Chernoff bound we showed in class, where $X = \sum_{i=1}^{n} X_i$ where X_i are indicator random variables:

$$\mathbf{Pr}[X \geq (1+\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{\mu}$$

Most of the interesting work in the proof was in analyzing $E[e^{tX}]$: as the X_i are independent, we could decompose it into a product of expectations that we then bounded. In this problem we will redo the analysis for the case where $X_i \sim \mathsf{Unif}(0,1)$ and see whether we can get a tighter bound.

(a) [3 pts] Calculate the bound the Chernoff bound above provides when each X_i is an indicator R.V. with $p_i = 1/2$ (i.e. it has equal probability of being 0 or 1).

(b) [2 pts] What is $E[X]$ when $X_i \sim \mathsf{Unif}(0,1)$?

(c) [10 pts] Compute $E[e^{tX}]$ in the case of uniform r.v.'s. Use this to give a Chernoff bound for $X_i \sim \mathsf{Unif}(0,1)$. You can set $t = \ln(1+\delta)$ as in the proof in class. For some $\delta > 0$ is this bound stronger or weaker than the bound from part 1? Why do you think this is the case?

Problem 2: Angle-Preserving JL (Har-Peled) [10 pts]

Show that the Johnson-Lindenstrauss lemma also $(1+\epsilon)$-preserves angles between triples of points (the target dimension may increase by a constant factor). Please give a short sketch of your solution. You can submit full proof details for bonus points. (Hint: What happens

to very skinny triangles if you just applied JL on the original points? Can you fix this by adding an extra triangle per angle with nicer structure that *stabilize* badly shaped triangles and let you argue angle preservation?)

Problem 3: Cover-Tree Insertion [10 pts]

Please refer to the reading for this problem: *cover tree reading*

Consider inserting elements into a Cover Tree. "Algorithm 2: Insert" from the reading on cover trees is written recursively. Give an example of a point set and insertion point in which the algorithm will return multiple levels up the recursion before executing step 3(b).