Recall the Chernoff bound we showed in class, where \( X = \sum_{i=1}^{n} X_i \) where \( X_i \) are indicator random variables:

\[
\Pr[X \geq (1 + \delta)\mu] \leq \left( \frac{e^{\delta}}{(1 + \delta)^{1+\delta}} \right)^\mu
\]

Most of the interesting work in the proof was in analyzing \( E[e^{tX}] \): as the \( X_i \) are independent, we could decompose it into a product of expectations that we then bounded. In this problem we will redo the analysis for the case where \( X_i \sim \text{Unif}(0, 1) \) and see whether we can get a tighter bound. For some \( \delta > 0 \) is this bound stronger or weaker than the bound from part 1? Why do you think this is the case?

(a) [3 pts] Calculate the bound the Chernoff bound above provides when each \( X_i \) is an indicator R.V. with \( p_i = 1/2 \) (i.e. it has equal probability of being 0 or 1).

(b) [2 pts] What is \( E[X] \) when \( X_i \sim \text{Unif}(0, 1) \)?

(c) [10 pts] Compute \( E[e^{tX}] \) in the case of uniform r.v.’s. Use this to give a Chernoff bound for \( X_i \sim \text{Unif}(0, 1) \). You can set \( t = \ln(1 + \delta) \) as in the proof in class. For some \( \delta > 0 \) is this bound stronger or weaker than the bound from part 1? Why do you think this is the case?

Problem 2: Angle-Preserving JL (Har-Peled) [10 pts]

Show that the Johnson-Lindenstrauss lemma also \((1 + \epsilon)\)-preserves angles between triples of points (the target dimension may increase by a constant factor). Please give a short sketch of your solution. You can submit full proof details for bonus points. (Hint: What happens
to very skinny triangles if you just applied JL on the original points? Can you fix this by adding an extra triangle per angle with nicer structure that stabilize badly shaped triangles and let you argue angle preservation?)

Problem 3: Cover-Tree Insertion [10 pts]

Please refer to the reading for this problem: cover tree reading

Consider inserting elements into a Cover Tree. “Algorithm 2: Insert” from the reading on cover trees is written recursively. Give an example of a point set and insertion point in which the algorithm will return multiple levels up the recursion before executing step 3(b).