15-853:Algorithms in the Real World

Nearest Neighbors in **High Dimensions**

- Representing Documents and Products as Sets, Set similarity
- Minhash for compact set signatures
- Locality sensitive hashing

"BigData"

Challenges

1. Presenting high dimensional objects compactly, so that they can be stored in the RAM and quickly compared for similarity.
   - Today: Min-hash signatures for sets

2. Finding similar items from a collection of high dimensional objects.
   - Today: Locality sensitive hashing based on min-hash

Material based largely on "Mining of Massive Datasets" book by Leskovec, Rajaraman and Ullman (available free for download!)

High Dimensional Data

Examples of high dimensional data:

Representing **documents** as vectors (or sets)

- "bag of words" (TF-IDF weighting)
- shingles (k-substrings)

"The course will cover both the theory behind the algorithms and case studies…"

➔ {the: 3, course: 1, will: 1, …}

➔ [0,0, …., 1 …, 3,0,0,… 1,0,..]

➔ For representing **sets**, only **binary values**

Extremely sparse, so we use *sparse vectors*

➔ [(118,1), (107872,1), (200938, 1) ….]

Note: In practice stop-words like "the" would be removed.

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"The course will cover both the theory behind the algorithms and case studies…"

➔ {the_course, he_course_ , e_course_w, … }

➔ or {the course will, course will cover, will cover both, …"

how many bytes/words for shingle length?

usually then hash down into fewer (32?) bits.

High Dimensional Data (cont.)

Collaborative Filtering

– represent movie as a vector of ratings by users

– represent product by binary vector x: x(j) = 1 if user j bought the item, 0 otherwise

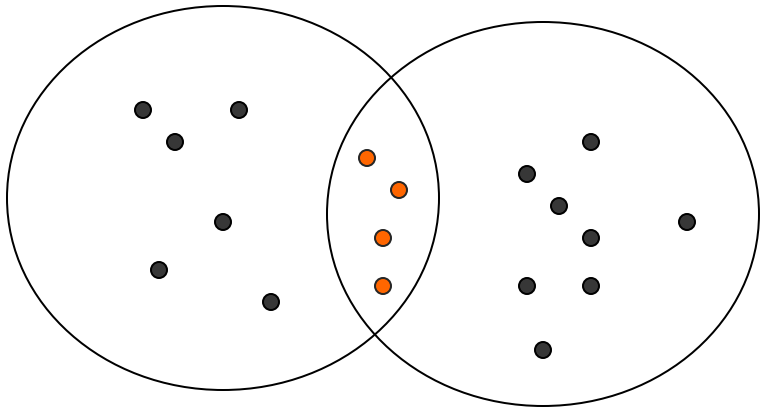
Applications of finding Similar (nearest) Items

- Filter duplicate docs in search engine
- Plagiarism, mirror pages
- Recommend similar products, movies

Defining Similarity

Similarity metric, “distance”, for **sets**

Jaccard similarity: $$\mathrm{SIM}(A, B) := \frac{A \cap B}{A \cup B}$$



4 common
18 total

SIM(A,B) = 4/18 = 2/9

Similarity-Preserving Signatures

Even sparse, the sets of words, shingles or users/ratings are too big to handle efficiently.

Goal: compute a "signature" for each set, so that similar documents have similar signatures (and dissimilar docs are unlikely to have similar signatures). (Note: "hashes" are one type of signature)

Trade-off: length of signature vs. accuracy

Could we use **cryptographic signatures?**

Characteristic Matrix of Sets

Element num	Set1	Set2	Set3	Set4
0	1	0	0	1
1	0	0	1	0
2	0	1	0	1
3	1	0	1	1
4	0	0	1	0
…				

Stored as a sparse matrix in practice.

Example from "Mining of Massive Datasets" book by Leskovec, Rajaraman and Ullman

Minhashing

Minhash(π) of a set is the number of the row (element) with first non-zero in the **permuted order π**.

Element num	Set1	Set2	Set3	Set4
1	0	0	1	0
4	0	0	1	0
0	1	0	0	1
3	1	0	1	1
2	0	1	0	1
…				

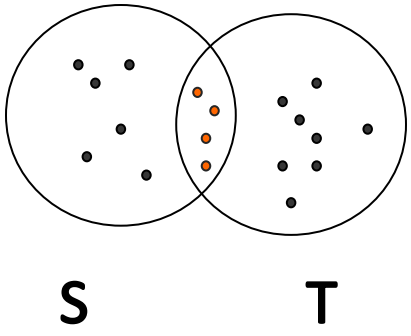
Π=(1,4,0,3,2)

Example from "Mining of Massive Datasets" book by Leskovec, Rajaraman and Ullman

Minhash and Jaccard similarity

Theorem:

P(minhash(S) = minhash(T)) = SIM(S,T)



Proof:

X = rows with 1 for both S and T

Y = rows with either S or T have 1, but not both

Z = rows with both 0

Probability that row of type X is before type Y in a random permuted order is _____

Minhash signature

Let $h_1, h_2, \ldots, h_n$ be different minhash functions (i.e different permutations).

Then signature for set S is:

$SIG(S) = [h_1(S), h_2(S), \ldots, h_n(S)]$

Now how to compute estimate of the Jaccard similarity between S and T using minhash-signatures?

SIM(S,T) ≈ ratio of equal elements of SIG(S) and SIG(T)

Approximating Minhashes

…. But storing huge permutations is also infeasible.

Solution: use a **random hash function** (for row number) to simulate a permutation.

Properties of random hashes?

We assume the # collisions is small vs. number of items.

Worked example (on blackboard)

Element num	Set1	Set2	Set3	Set4	x + 1 mod 5	3x +1 mod 5
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3
…						

Signature matrix

	Set1	Set2	Set3	Set4
H1	∞	∞	∞	∞
H2	∞	∞	∞	∞

Algorithm

For each row r = 0, 1, …, N-1 of the characteristic matrix:

1. Compute $h_1(r)$, $h_2(r)$, …, $h_n(r)$
2. For each column c:
    1. If column c has 0 in row r, do nothing
    2. Otherwise, for each i = 1,2, …, n set SIG(i, c) to be $\min(h_i(r), SIG(i, c))$

Note: in practice we need to only iterate through the non-zero elements.

LOCALITY SENSITIVE HASHING USING MINHASH

Nearest Neighbors

Assume that we construct a 1,000 byte minhash signature for each document.

Million documents can now fit into 1 gigabyte of RAM.

But how much does it cost to find the nearest neighbor of a document?

- Brute force: N comparisons.

→ Need a way to reduce the number of comparisons.

LSH requirements

A hash function will divide input into large number of **buckets.** To find nearest neighbors for a query item q, we want to only compare with items in the bucket hash(q): "candidates".

If two A and B are similar, we want the probability that hash(A) = hash(B) be high.

- *False positives*: sets that are not similar, but are hashed into same bucket.
- *False negatives*: sets that are similar, but hashed into different buckets.

LSH based on minhash

(do not get confused about the different “hashes”)

Idea:

divide the signature matrix rows into **b** bands of **r** rows

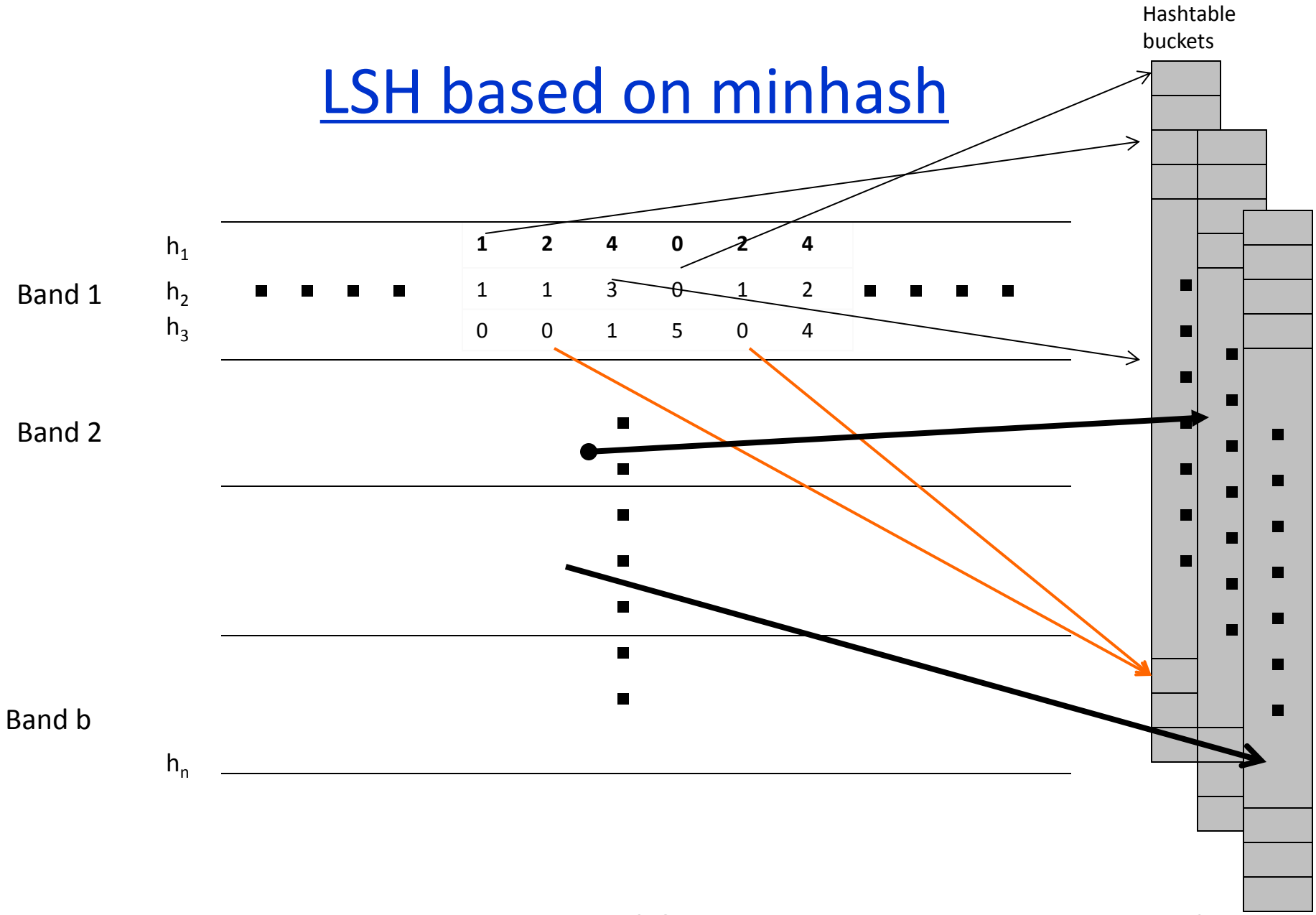
hash the columns in each band with a basic hash-function

→ each band divided to buckets [i.e a hashtable for each band]

If sets S and T have same values in a band, they will be hashed into the same bucket in that band.

For nearest-neighbor, the candidates are the items in the same bucket as query item, in each band.

LSH based on minhash



Analysis

Consider probability that we find T with query document Q

Let

s = SIM(Q,T) = P{ $h_i(Q) = h_i(T)$ }

b = # of bands

r = # rows in one band

What is the probability that rows of signature matrix agree for columns Q and T in one band?

Analysis

s = SIM(Q,T)
b = # of bands
r = # rows in one band

Probability that Q and T agree on all r rows in a band

s^r

Probability that disagree on at least one row

$1 - s^r$

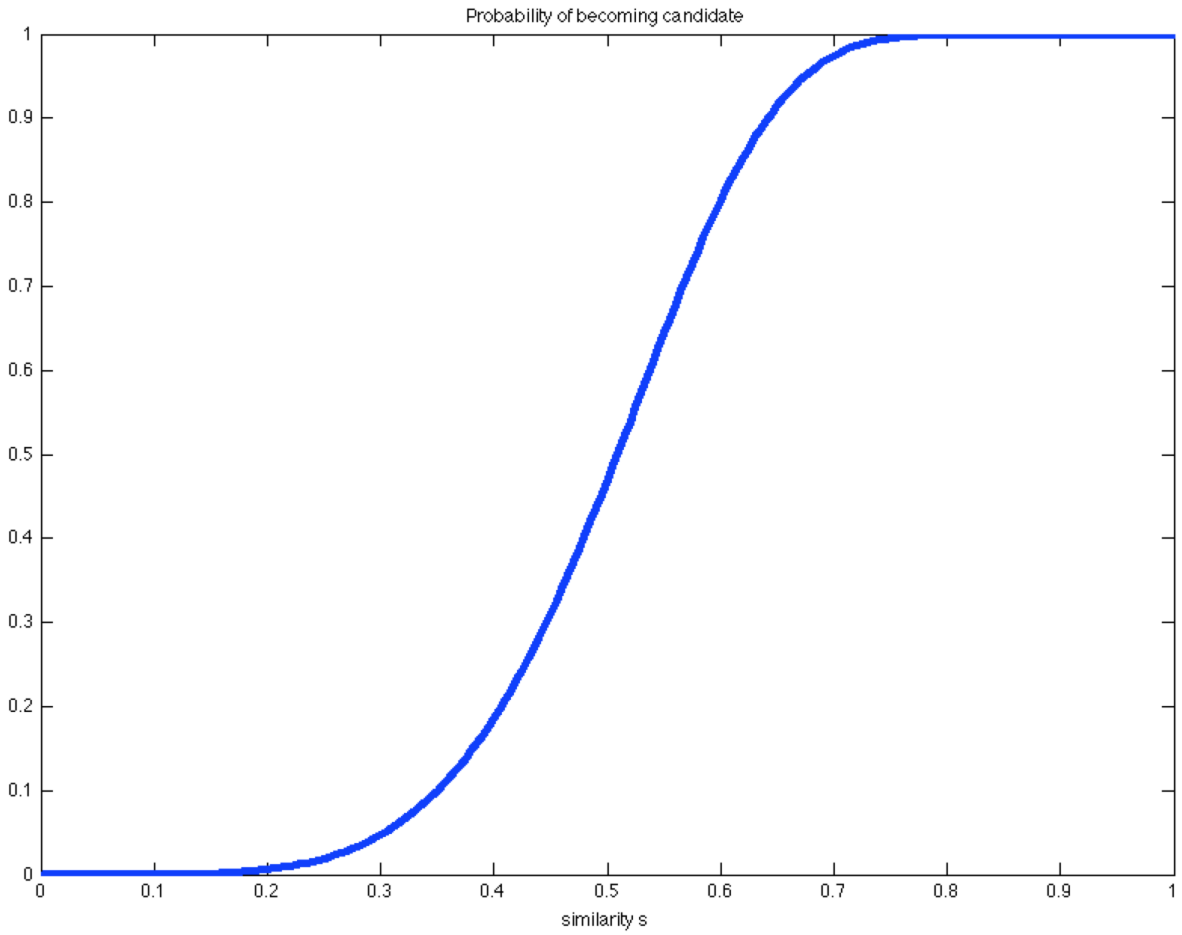
Probability that signatures do not agree on all the bands:

$(1 - s^r)^b$

Probability that T will be chosen as candidate: ____

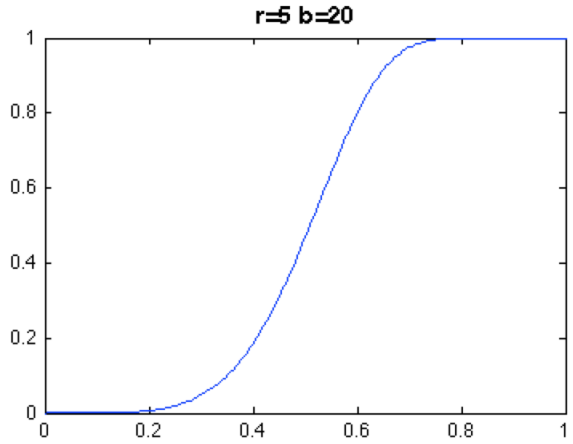
S-curve

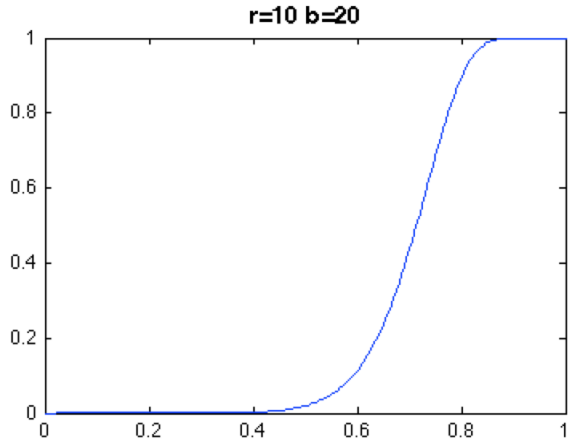
r = 5
b = 20

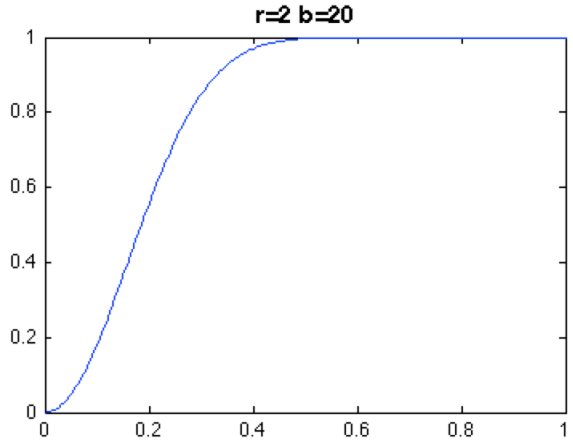


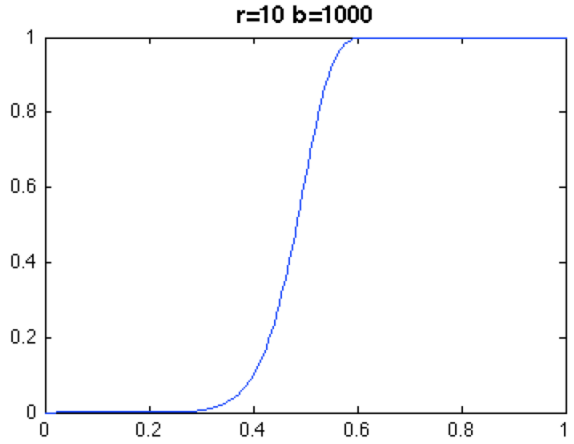
S-curves

r and b are parameters of the system: trade-offs?









Summary

To build a system that quickly finds similar documents from a corpus:

1. Decide a vector presentation of your documents
   (bag of words, shingles, etc…)
2. Generate minhash signature matrix for the corpus.
3. Divide signature matrix into bands
4. Store each band-column into a hashtable
5. To find similar documents, compare to candidate documents for each band only in the same bucket
   (using minhash signatures or the docs themselves) .

More About Locality Sensitive Hashing

Active research area.

Different distance metrics and compatible locality sensitive hash functions:

- Euclidean distance → random projections
- Cosine distance
- Edit distance (strings)
- Hamming distance

On the board…

More About Locality Sensitive Hashing

Leskovec, Rajaraman, Ullman: Mining of Massive Datasets (available for download)

CACM technical survey article by Andoni and Indyk.