15-499:Algorithms and Applications

Data Compression III
- Lempel-Ziv algorithms
- Burrows-Wheeler

Compression Outline

Introduction: Lossy vs. Lossless, Benchmarks, …
Information Theory: Entropy, etc.
Probability Coding: Huffman + Arithmetic Coding
Applications of Probability Coding: PPM + others
Lempel-Ziv Algorithms:
- LZ77, gzip,
- LZ78, compress

Other Lossless Algorithms: Burrows-Wheeler
Lossy algorithms for images: JPEG, MPEG, ...
Compressing graphs and meshes: BBK

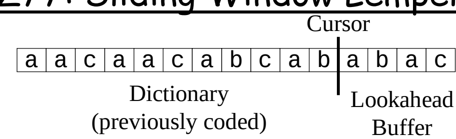
Lempel-Ziv Algorithms

LZ77 (Sliding Window)
- **Variants**: LZSS (Lempel-Ziv-Storer-Szymanski)
- **Applications**: `gzip`, Squeeze, LHA, PKZIP, ZOO

LZ78 (Dictionary Based)
- **Variants**: LZW (Lempel-Ziv-Welch), LZC
- **Applications**: `compress`, GIF, CCITT (modems), ARC, PAK

Traditionally LZ77 was better but slower, but the gzip version is almost as fast as any LZ78.

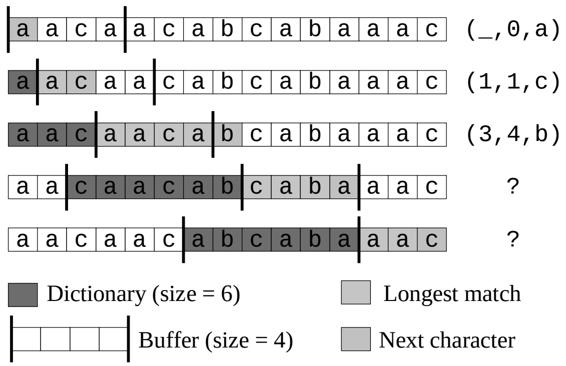
LZ77: Sliding Window Lempel-Ziv



Dictionary and **buffer** "windows" are fixed length and slide with the **cursor**

Repeat:
- Output ***(p, l, c)*** where
  - ***p*** = position of the longest match in the dictionary (relative to the cursor)
  - ***l*** = length of longest match
  - **c** = next char in buffer beyond longest match
- Advance window by ***l*** + 1

LZ77: Example



LZ77 Decoding

Decoder keeps same dictionary window as encoder.

For each message it looks it up in the dictionary and inserts a copy

What if *l* > p? (only part of the message is in the dictionary.)

E.g. dict = `abcd`, codeword = `(2,9,e)`

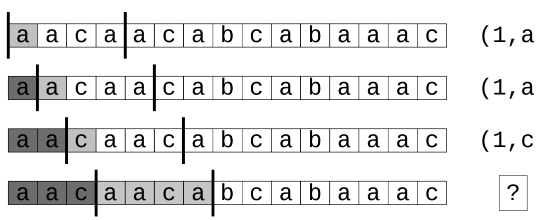
What does this give?

LZ77 Optimizations used by `gzip`

LZSS: Output one of the following two formats

`(0, position, length)` or **`(1,char)`**

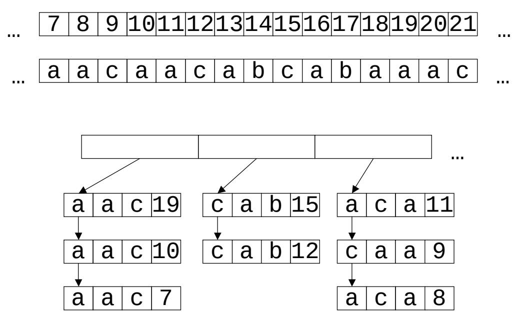
Uses the second format if length < 3.



Optimizations used by `gzip` (cont.)

1. Huffman code the positions, lengths and chars
2. Non greedy: possibly use shorter match so that next match is better
3. Use a hash table to store the dictionary.
   - Hash keys are all strings of length 3 in the dictionary window.
   - Find the longest match within the correct hash bucket.
   - Puts a limit on the length of the search within a bucket.
   - Within each bucket store in order of position

The Hash Table



Theory behind LZ77

Sliding Window LZ is Asymptotically Optimal [Wyner-Ziv,94]

Will compress long enough strings to the source entropy as the window size goes to infinity.

$$H_n = \sum_{X \in A^n} p(X) \log \frac{1}{p(X)}$$

$$H = \lim_{n \to \infty} H_n$$

Uses logarithmic code (e.g. gamma) for the position.

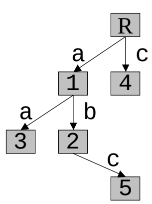
Problem: "long enough" is really **really** long.

LZ78: Dictionary Lempel-Ziv

Like LZ77, but "dictionary" is maintained differently

The **dictionary** maintains a mapping of **<u>words</u>** to ***<u>ids</u>***

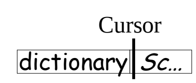
Typically stored as a trie, *e.g.*:



Word	*id*
a	1
aa	3
ab	2
abc	5
c	4

LZ78: Encoding and Decoding

Keep a cursor as in LZ77



<u>Repeat</u>:

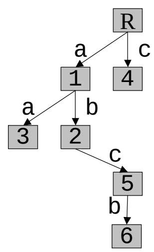
- Output (***id***, ***c***) where
  - ***id*** = The *id* of the longest match *S* found in the dictionary
  - ***c*** = next char in buffer beyond longest match
- Add the string *Sc* to the dictionary with a new *id*
- Move the cursor past *S* and *c*

<u>Decoding</u> keeps the same dictionary and looks up *ids*

LZ78: Coding Example

	Output
a a b a a c a b c a b c b	(0,a)
a a b a a c a b c a b c b	(1,b)
a a b a a c a b c a b c b	(1,a)
a a b a a c a b c a b c b	(0,c)
a a b a a c a b c a b c b	?
a a b a a c a b c a b c b	?

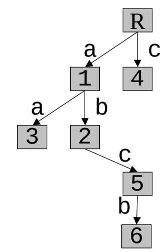
Dict.



LZ78: Decoding Example

Input	
(0,a)	a
(1,b)	a a b
(1,a)	a a b a a
(0,c)	a a b a a c
(2,c)	a a b a a c a b c
(5,b)	a a b a a c a b c a b c b

Dict.



LZW (Lempel-Ziv-Welch) [Welch84]

Don't send the extra character *c*, but still add *Sc* to the dictionary.

Initialize the dictionary with all possible character values. For bytes, the dictionary starts with 256 entries.

Decoder can only add *Sc* to the dictionary after decoding the next word since it does not know c.

LZW : Possible Problem

The decoder is one step behind the encoder since it does not know *c*.

Consider a string …*SSc*… where *S[0] = c*

The encoder:

- Outputs the id_1 for *S* and enters *Sc* into dictionary creating a new id_2 for Sc.
- Outputs the id_2 for *Sc*

The decoder:

- Inputs id_1, looks it up and outputs *S*
- Inputs id_2, looks it up, but not in the dictionary

What do we do?

LZW: Encoding Example

													Output	Dict.
a	a	b	a	a	c	a	b	c	a	b	c	b	112	256=aa
a	**a**	b	a	a	c	a	b	c	a	b	c	b	112	257=ab
a	a	**b**	a	a	c	a	b	c	a	b	c	b	113	258=ba
a	a	b	**a**	**a**	c	a	b	c	a	b	c	b	256	259=aac
a	a	b	a	a	**c**	a	b	c	a	b	c	b	114	260=ca
a	a	b	a	a	c	**a**	**b**	c	a	b	c	b	257	261=abc
a	a	b	a	a	c	a	b	**c**	**a**	b	c	b	260	262=cab

LZ78 and LZW issues

What happens when the dictionary gets too large?

- Throw the dictionary away when it reaches a certain size (used in GIF)
- Throw the dictionary away when it is no longer effective at compressing (used in unix `compress`)
- Throw the least-recently-used (LRU) entry away when it reaches a certain size (used in BTLZ, the British Telecom standard)

LZW: Decoding Example

Input														Dict
112	**a**	a	b	a	a	c	a	b	c	a	b	c	b	
112	**a**	**a**	b	a	a	c	a	b	c	a	b	c	b	256=aa
113	a	**a**	**b**	a	a	c	a	b	c	a	b	c	b	257=ab
256	a	a	**b**	**a**	**a**	c	a	b	c	a	b	c	b	258=ba
114	a	a	b	**a**	**a**	**c**	a	b	c	a	b	c	b	259=aac
257	a	a	b	a	a	**c**	**a**	**b**	c	a	b	c	b	260=ca
260	a	a	b	a	a	c	**a**	**b**	**c**	**a**	b	c	b	261=abc

Lempel-Ziv Algorithms Summary

Both LZ77 and LZ78 and their variants keep a "dictionary" of recent strings that have been seen.

The differences are:

- How the dictionary is stored
- How it is extended
- How it is indexed
- How elements are removed

Lempel-Ziv Algorithms Summary (II)

Adapts well to changes in the file (e.g. a Tar file with many file types within it).

Initial algorithms did not use probability coding and performed poorly in terms of compression. More modern versions (e.g. gzip) do use probability coding as "second pass" and compress much better.

The algorithms are becoming outdated, but ideas are used in many of the newer algorithms.

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Applications of Probability Coding: PPM + others
Lempel-Ziv Algorithms: LZ77, gzip, compress, …
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- ACB

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Burrows -Wheeler

Currently best "balanced" algorithm for text

Breaks file into fixed-size blocks and encodes each block separately.

For each block:

- Sort each character by its full context. This is called the **block sorting transform**.
- Use **move-to-front transform** to encode the sorted characters.

The ingenious observation is that the decoder only needs the sorted characters and a pointer to the first character of the original sequence.

Burrows Wheeler: Example

Let's encode: $d_1e_2c_3o_4d_5e_6$

We've numbered the characters to distinguish them.

Context "wraps" around.

Context	Char		Context	Output	
$ecode_6$	d_1	**Sort Context** →	$dedec_3$	o_4	
$coded_1$	e_2		$coded_1$	e_2	
$odede_2$	c_3		$decod_5$	e_6	
$dedec_3$	o_4		$odede_2$	c_3	
$edeco_4$	d_5		$ecode_6$	d_1	⇐
$decod_5$	e_6		$edeco_4$	d_5	

Burrows-Wheeler (Continued)

Theorem: *After sorting, equal valued characters appear in the same order in the output as in the most significant position of the context.*

Proof sketch: Since the chars have equal value in the most-significant-position of the context, they will be ordered by the rest of the context, *i.e.* the previous chars. This is also the order of the output since it is sorted by the previous characters.

Context	Output
dedec$_3$	o$_4$
coded$_1$	e$_2$
decod$_5$	e$_6$
odede$_2$	c$_3$
ecode$_6$	d$_1$
edeco$_4$	d$_5$

Burrows-Wheeler: Decoding

Consider dropping all but the last character of the context.
- What follows the **a** with an arrow?
- What follows the first **b**?
- What is the whole string?

Context	**Output**	
a	c	
a	b	
a	b	
b	a	
b	a	⇐
c	a	

Burrows-Wheeler: Decoding

What about now?

Can also use the "rank".

The "rank" is the position of a character if it were sorted using a stable sort.

Output		**Rank**
c	⇐	6
a		1
b		4
b		5
a		2
a		3

Burrows-Wheeler Decode

```
Function BW_Decode(In, Start, n)
  S = MoveToFrontDecode(In,n)
  R = Rank(S)
  j = Start
  for i=1 to n do
     Out[i] = S[j]
     j = R[j]
```

Rank gives position of each char in sorted order.

Decode Example

S	Sort(S)	Rank(S)			Out	
o_4	c_3	6		e_6	d_1	⇐
e_2	d_1	4		d_1	e_2	
e_6	d_5	5		e_2	c_3	
c_3	e_2	1		c_3	o_4	
d_1	e_6	2 ⇐		o_4	d_5	
d_5	o_4	3		d_5	e_6	

Overview of Text Compression

PPM and **Burrows-Wheeler** both encode a single character based on the immediately preceding context.

LZ77 and **LZ78** encode multiple characters based on matches found in a block of preceding text

Can you **mix these ideas**, *i.e.,* code multiple characters based on immediately preceding context?

- **BZ** does this, but they don't give details on how it works – current best compressor
- **ACB** also does this – close to best

ACB (Associate Coder of Buyanovsky)

Keep dictionary sorted by context (the last character is the most significant)

- Find longest match for context
- Find longest match for contents
- Code
  - Distance between matches in the sorted order
  - Length of contents match

Has aspects of Burrows-Wheeler, and LZ77

Context	Contents
	decode
dec	ode
d	ecode
decod	e
de	code
deco	de