15-853:Algorithms in the Real World

Parallelism: Lecture 2

Parallel techniques and algorithms

-Working with collections

-Divide and conquer

Parallel Techniques

Some common themes in "Thinking Parallel"

1. Working with collections.
   - map, selection, reduce, scan, collect
2. Divide-and-conquer
   - Even more important than sequentially
   - Merging, matrix multiply, FFT, …
3. Contraction
   - Solve single smaller problem
   - List ranking, graph contraction
4. Randomization
   - Symmetry breaking and random sampling

Working with Collections

reduce ⊙ [a, b, c, d, …
  = a ⊙ b ⊙ c ⊙ d + …

scan ⊙ ident [a, b, c, d, …
  = [ident, a, a ⊙ b, a ⊙ b ⊙ c, …

sort compF A

collect [(2,a), (0,b), (2,c), (3,d), (0,e), (2,f)]
  = [(0, [b,e]), (2,[a,c,f]), (3,[d])]

Example of scan: parentheses matching

The parentheses matching problem:

- Check if a set of a single kind of parentheses match
- E.g. (()(()())) matches (((()()))(()() does not
- Easy to do serially by scanning left to right keeping a counter.
- How do we do this in parallel

Example of scan: parentheses matching

The parentheses matching using a scan:

```
function parenthesesMatch(S) =
let
  A = {if c == '( then 1 else -1: c in S};
  Sums = scan(add,0,A);
in
  (reduce(min,Sums) >= 0)
```

Can also do it with a map and reduce.

Example of Collect: Building an Index

Problem: Given a set of documents each a string, compute an index that maps words to documents.

[(1,"this is the first document"),
(2,"this is the second"),
(3,"the third"),
(4,"and the fourth")]

[("and",[4]),("first",[1]),…,("is",[1,2]), …,
("the",[1,2,3,4]),("this",[1,2]),("third",[3])]

Example of Collect: Building an Index

Problem: Given a set of documents each with a sequence of words, compute an index that maps words to documents.

```
function makeIndex D =
  let
    a = flatten({{(w,i) : w in wordify(d)}
                 : (i,d) in D})
  in collect(a);
```


MapReduce

```
function mapReduce(MAP,REDUCE,documents) =
 let
    temp = flatten({MAP(d) : d in documents});
 in flatten({REDUCE(k,vs) : (k,vs) in collect(temp)});

function mapRed(M,R) = (D => mapReduce(M,R,D));

wordcount = mapReduce(d => {(w,1) : w in wordify(d)},
                      (w,c) => [(w,sum(c))]);

wordcount(["this is is document 1",
           "this is document 2"]);
```

Technique 2: Divide-And-Conquer

- Merging
- Matrix multiplication
- Matrix inversion
- FFT
- K-d trees

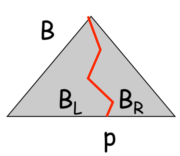
Example: Merging

```
Merge(nil,l2) = l2
Merge(l1,nil) = l1
Merge(h1::t1, h2::t2) =
    if (h1 < h2) h1::Merge(t1,h2::t2)
    else h2::Merge(h1::t1,t2)
```

What about in parallel?

The Split Operation

```
fun split (p, empty) = (empty ,empty)
  | split (p, node(v, L, R)) =
    if p < v then
      let val (L1 ,R1) = split(p ,L)
      in (L1,node(v, R1, R)) end
    else
      let val (L1,R1) = split(p ,R)
      in (node (v, L, L1), R1) end;
```

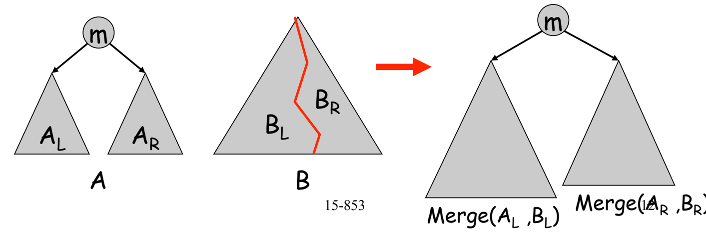


Merging

```
Merge(A,B) =
  let
    Node(A_L, m, A_R) = A
    (B_L ,B_R) = split(B, m)
  in
    Node(Merge(A_L,B_L), m, Merge(A_R,B_R))
```

Span = $O(\log^2 n)$
Work = $O(n)$

Merge in parallel



MergeSort

```
function mergeSort(S) =
if (#S < 2) S
else merge(mergesort(S[0:#S/2]),
           mergesort(S[#S/2:#S]))
```

W(n) = 2 W(n/2) + O(n) = O(n log n)

What about the span?

Matrix Multiplication

```
Fun A*B {
  if #A < k then baseCase..
  C11 = A11*B11 + A12*B21
  C12 = A11*B12 + A12*B22
  C21 = A21*B11 + A22*B21
  C22 = A21*B12 + A22*B22
  return C
}
```

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\begin{aligned} W_*(n) &= 8W(n/2) + O(n^2) \\ &= O(n^3) \end{aligned} \qquad \begin{aligned} D(n) &= D(n/2) + O(1) \\ &= O(\log n) \end{aligned}$$

$$Parallelism = \frac{W}{D} = O\left(\frac{n^3}{\log n}\right)$$

Matrix Inversion

```
fun invert(M) {
  if small baseCase
  D^-1 = invert(D)
  S   = A - BD^-1C
  S^-1 = invert(S)
  E = S^-1
  F = S^-1BD^-1
  G = -D^-1CS^-1
  H = D^-1 + D^-1CS^-1BD^-1}
```

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad M^{-1} = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{aligned} W(n) &= 2W(n/2) + 6W_*(n/2) \\ &= O(n^3) \end{aligned} \qquad \begin{aligned} D(n) &= 2D(n/2) + 6D_*(n/2) \\ &= O(n) \end{aligned}$$

$$Parallelism = \frac{W}{D} = O(n^2)$$

Fourier Transform

```
function fft(a,w) =
if #a == 1 then a
else
  let r = {fft(b, even_elts(w)):
           b in [even_elts(a),odd_elts(a)]}
  in {a + b * w : a in r[0] ++ r[0];
                  b in r[1] ++ r[1];
                  w in w};
```

$$\begin{aligned} W(n) &= 2W(n/2) + O(n) \\ &= O(n \log n) \end{aligned} \qquad \begin{aligned} D(n) &= D(n/2) + O(1) \\ &= O(\log n) \end{aligned}$$

$$Parallelism = \frac{W}{D} = O(n)$$

Spatial Decompositions: Revisited

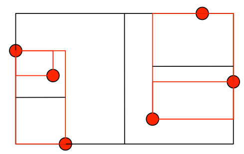
Typically consist of:

- Split the data points into some constant number of parts. This is similar to the selection in Quicksort.
- Recursively subdivide within each part.

Both of these are easy to parallelize, but problematic if highly imbalanced.

Callahan-Kosaraju: Build Tree

Function Tree(P)
if |P| = 1 then **return** leaf(P)
else
  d_{max} = dimension of l_{max}
  P_1, P_2 = split P along d_{max} at midpoint
  Return Node(Tree(P_1),Tree(P_2), l_{max})



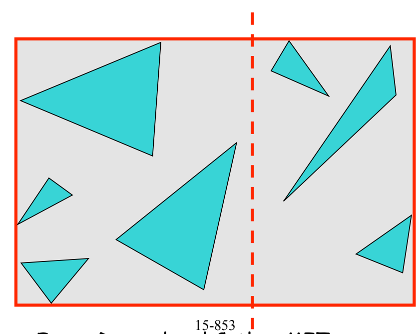
KK: Generating the Realization

function wsr(T)
if leaf(T) **return** ∅
else return wsr(left(T)) ∪ wsr(right(T))
  ∪ wsrP(left(T),right(T))

function wsrP(T_1, T_2)
if wellSep(T_1,T_2) **return** {(T_1,T_2)}
else if $l_{max}(T_1) > l_{max}(T_2)$ **then**
  return wsrP(left(T_1), T_2) ∪ wsrP(right(T_1), T_2)
else
  return wsrP(T_1, left(T_2)) ∪ wsrP(T_1, right(T_2))

Bounding Volume Hierarchy: Top Down

Find bounding box of objects
Split objects into two groups
Recurse



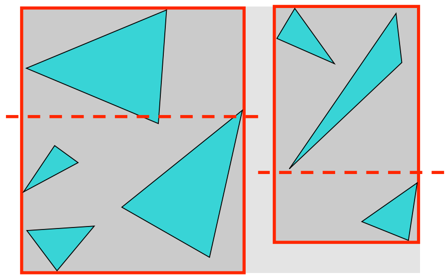
From: Durand and Cutler, MIT

Bounding Volume Hierarchy

Find bounding box of objects

Split objects into two groups

Recurse



Durand and Cutler

Parallel Techniques

Some common themes in “Thinking Parallel”

1. Working with collections.
   - map, selection, reduce, scan, collect
2. Divide-and-conquer
   - Even more important than sequentially
   - Merging, matrix multiply, FFT, …
3. Contraction
   - Solve single smaller problem
   - List ranking, graph contraction
4. Randomization
   - Symmetry breaking and random sampling