15-853:Algorithms in the Real World

Suffix Trees for Compression

Lempel-Ziv Algorithms

LZ77 (Sliding Window)

Variants: LZSS (Lempel-Ziv-Storer-Szymanski)

Applications: `gzip`, Squeeze, LHA, PKZIP, ZOO

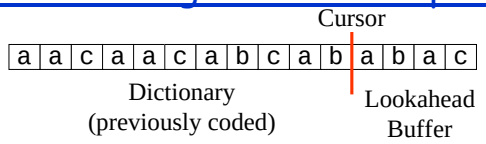
LZ78 (Dictionary Based)

Variants: LZW (Lempel-Ziv-Welch), LZC

Applications: `compress`, GIF, CCITT (modems), ARC, PAK

Traditionally LZ77 was better but slower, but the gzip version is almost as fast as any LZ78.

LZ77: Sliding Window Lempel-Ziv



Dictionary and **buffer** "windows" are fixed length and slide with the **cursor**

Repeat:

Output **(*p*, *l*, c)** where

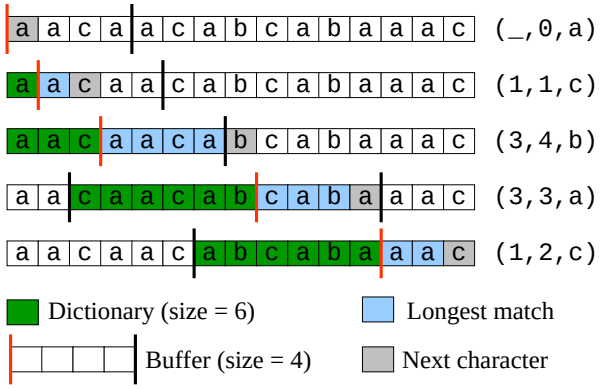
p = position of the longest match that starts in the dictionary (relative to the cursor)

l = length of longest match

c = next char in buffer beyond longest match

Advance window by ***l*** + 1

LZ77: Example



LZ77 Decoding

Decoder keeps same dictionary window as encoder.

For each message it looks it up in the dictionary and inserts a copy at the end of the string

What if $l > p$? (only part of the message is in the dictionary.)

E.g. dict = abcd, codeword = (2,9,e)

- Simply copy from left to right

```
for (i = 0; i < length; i++)
  out[cursor+i] = out[cursor-offset+i]
```

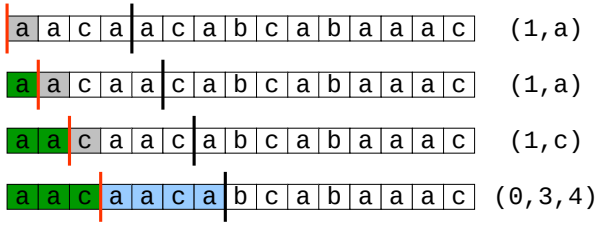
- Out = abcdcdcdcdce

LZ77 Optimizations used by gzip

LZSS: Output one of the following two formats

(0, position, length) or **(1,char)**

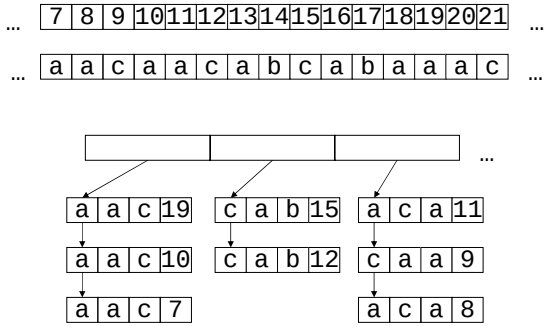
Uses the second format if length < 3.



Optimizations used by gzip (cont.)

1. Huffman code the positions, lengths and chars
2. Non greedy: possibly use shorter match so that next match is better
3. Use a hash table to store the dictionary.
   - Hash keys are all strings of length 3 in the dictionary window.
   - Find the longest match within the correct hash bucket.
   - Puts a limit on the length of the search within a bucket.
   - Within each bucket store in order of position

The Hash Table



Using Suffix Trees

Recall that at each stage, we output a pair (p_i, l_i) where $S[p_i .. p_i+l_i] = S[i .. i+l_i]$

Find all pairs (p_i, l_i) in linear time

Construct a suffix tree for S

Label each internal node with the minimum of labels of all leaves below it – this is the first place in S where it occurs. Call this label c_v.

For every i, search for the string S[i .. m] stopping just before c_v, i. This gives us l_i and p_i.

Theory behind LZ77

Sliding Window LZ is Asymptotically Optimal [Wyner-Ziv,94]

Will compress long enough strings to the source entropy as the window size goes to infinity.

$$H_n = \sum_{X \in A^n} p(X) \log \frac{1}{p(X)}$$

$$H = \lim_{n \to \infty} H_n$$

Uses logarithmic code (e.g. gamma) for the position.

Problem: "long enough" is really **really** long.

Lempel-Ziv Algorithms Summary

Adapts well to changes in the file (e.g. a Tar file with many file types within it).

Initial algorithms did not use probability coding and performed poorly in terms of compression. More modern versions (e.g. gzip) do use probability coding as "second pass" and compress much better.

The algorithms are becoming outdated, but ideas are used in many of the newer algorithms.

Burrows -Wheeler

Currently near best "balanced" algorithm for text:
(**Archive Comparison Test** , compressia is BW)

Breaks file into fixed-size blocks and encodes each block separately.

For each block:

- Sort each character by its full context. This is called the **block sorting transform**.
- Use **move-to-front transform** to encode the sorted characters.

The ingenious observation is that the decoder only needs the sorted characters and a pointer to the first character of the original sequence.

Burrows Wheeler: Example

Let's encode: $d_1e_2c_3o_4d_5e_6$

We've numbered the characters to distinguish them.

Context "wraps" around. Last char is most significant.

Context	Char
$ecode_6$	d_1
$coded_1$	e_2
$odede_2$	c_3
$dedec_3$	o_4
$edeco_4$	d_5
$decod_5$	e_6

Sort Context →

Context	Output
$dedec_3$	o_4
$coded_1$	e_2
$decod_5$	e_6
$odede_2$	c_3
$ecode_6$	d_1 ⇐
$edeco_4$	d_5

Burrows-Wheeler (Continued)

Theorem: *After sorting, equal valued characters appear in the same order in the output as in the most significant position of the context.*

Proof sketch: Since the chars have equal value in the most-significant-position of the context, they will be ordered by the rest of the context, *i.e.* the previous chars. This is also the order of the output since it is sorted by the previous characters.

Context	Output
[redacted] c_3	o_4
[redacted] d_1	e_2
[redacted] d_5	e_6
[redacted] e_2	c_3
[redacted] **e_6**	d_1
[redacted] **o_4**	d_5

Burrows-Wheeler: Decoding

Consider dropping all but the last character of the context.

- What follows the underlined **a** ?
- What follows the underlined **b**?
- What is the whole string?

Answer: `b, a, abacab`

Context	**Output**
a	c
a	**b**
a	b
b	a
b	**a** ⇐
c	a

Burrows-Wheeler: Decoding

What about now?

Answer: `cabbaa`

Can also use the "rank".

The "rank" is the position of a character if it were sorted using a stable sort.

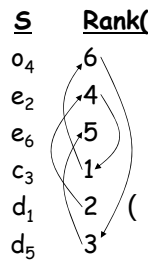
Context	**Output**	**Rank**
a	c ⇐	6
a	a	1
a	b	4
b	b	5
b	a	2
c	a	3

Burrows-Wheeler Decode

```
Function BW_Decode(In, Start, n)
   S = MoveToFrontDecode(In,n)
   R = Rank(S)
   j = Start
   for i=1 to n do
        Out[i] = S[j]
        j = R[j]
```

Rank gives position of each char in sorted order.

Decode Example



	Out
e_6	d_1 ⇐
d_1	e_2
e_2	c_3
c_3	o_4
o_4	d_5
d_5	e_6

BW Transform

Need to sort based on all prefixes:

Naïve : O(m x n log n)

How can we sort much faster?