

# 15-853: Algorithms in the Real World

Computational Biology III  
- Multiple Sequence Alignment

## Multiple Alignment

```
A C T _ G T A
A C A C G T T
A G T G _ T A
C C _ G C T A
```

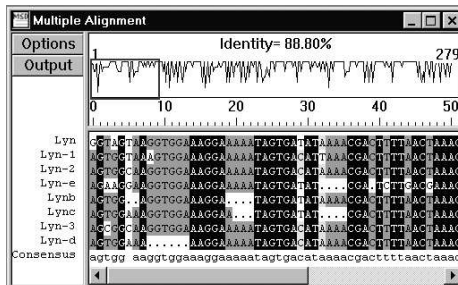
**Goal:** match the "maximum" number of aligned pairs of symbols.

**Applications:**

- Assembling multiple noisy reads of fragments of sequences
- Finding a canonical among members of a family and studying how the members differ

The problem is NP-hard

## Example Output



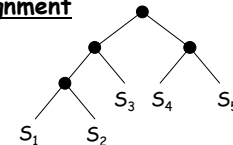
Output from typical multiple alignment software DNAMAN (using ClustalW)

## Scoring Multiple Alignments

1. **Distance from consensus  $S_c$ :**  $D = \sum_{S_i \in S} D(S_i, S_c)$

2. **Pairwise distances:**  $D = \sum_{S_i \in S} \sum_{S_j \in S / S_i} D(S_i, S_j)$

3. **Evolutionary Tree Alignment**



$D = D(S_1, S_2) + D(S_4, S_5) + D(S_{12}, S_3) + D(S_{123}, S_{45})$

## Approaches

**Dynamic programming:** optimal, but takes time that is exponential in  $p$

**Center Star Method:** approximation

**Clustering Methods:** also called iterative pairwise alignment. Typically an approximation. Many variants, many software packages

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## Using Dynamic Programming

For  $p$  sequences of length  $n$  we can fill in a  $p$ -dimensional array in  $n^p$  time and space.

For example for  $p = 3$ :

$$D_{ijk} = \min \begin{cases} D_{i-1,j-1,k-1} + d(a_i, b_j, c_k) \\ D_{i-1,j-1,k} + d(a_i, b_j, \_) \\ D_{i-1,j,k} + d(a_i, \_) \\ \dots \end{cases} \quad 7 \text{ cases}$$

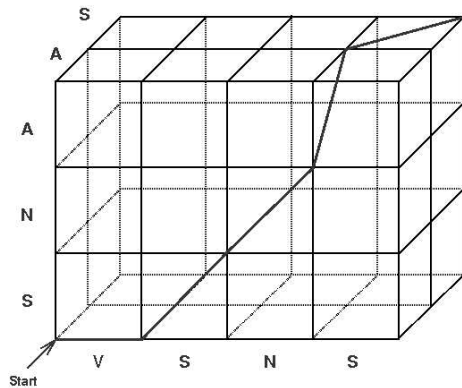
where  $d(a,b,c) = d(a,b) + d(b,c) + d(a,c)$  assuming the pairwise distance metric.

Takes time exponential in  $p$ . Perhaps OK for  $p = 3$

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## Example



```
V S N - S
- S N A -
- - - A S
```

## Optimization

As in the case of pairwise alignment we can view the array as a graph and find shortest paths.

Used in a program called MSA.

Can align 6 strings consisting of 200 bp each in a "practical" amount of time.

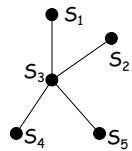
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## Center Star Method

1. Find  $S_r \in S$  minimizing  $\sum_{S_i \in S/S_r} D(S_i, S_r)$
2. Add remaining sequences  $S/S_r$  one by one so alignment of each is optimal wrt  $S_r$ .  
Add spaces if needed

**Time:**  $O(p^2n^2)$

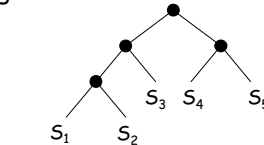


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## Using Clustering

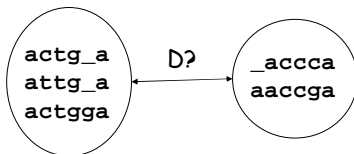
1. Compute  $D(S_i, S_j)$  for all pairs
2. Bottom up cluster
  - I. All sequences start as their own cluster
  - II. Repeat
    - a) find the two "closest" clusters and join them into one
    - b) Find best alignment of the two clusters being joined



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## Distances between Clusters



Could use difference between consensus.  
A popular technique is called the "Unweighted Pair-Group Method using arithmetic Averages" (UPGMA).  
It takes the average of all distances among the two clusters.  
Implemented in Clustal and Pileup

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## Summary of Matching

### Types of matching:

- **Global:** align two sequences A and B
- **Local:** align A with any part of B
- **Multiple:** align k sequences (NP-complete)

### Cost models

- **LCS and MED**
- **Scoring matrices:** Blosum, PAM
- **Gap cost:** affine, general

### Methods

- **Dynamic programming:** many optimizations
- **"Fingerprinting":** hashing of small seqs. (approx.)
- **Clustering:** for multiple alignment (approx.)

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