15-853:Algorithms in the Real World

Data Compression IV.5

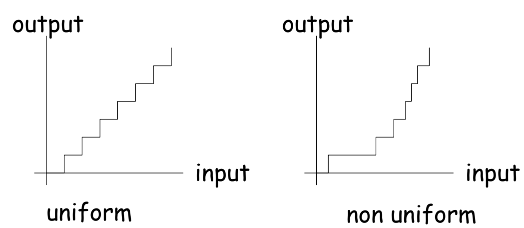
Compression Outline

Introduction: Lossy vs. Lossless, Benchmarks, …
Information Theory: Entropy, etc.
Probability Coding: Huffman + Arithmetic Coding
Applications of Probability Coding: PPM + others
Lempel-Ziv Algorithms: LZ77, gzip, compress, …
Other Lossless Algorithms: Burrows-Wheeler
Lossy algorithms for images: JPEG, MPEG, ...
- Scalar and vector quantization
- JPEG and MPEG

Compressing graphs and meshes: BBK

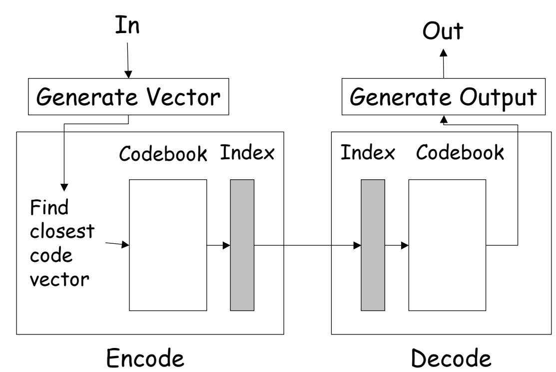
Scalar Quantization

Quantize regions of values into a single value:



Can be used to reduce # of bits for a pixel

Vector Quantization



Vector Quantization

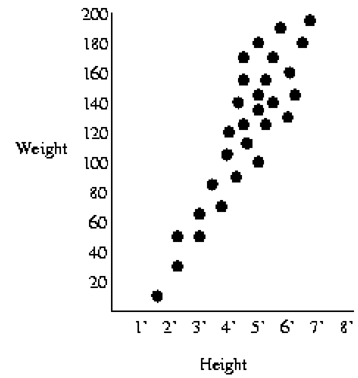
What do we use as vectors?

- Color (Red, Green, Blue)
  - Can be used, for example to reduce 24bits/pixel to 8bits/pixel
  - Used in some terminals to reduce data rate from the CPU (colormaps)
- K consecutive samples in audio
- Block of K pixels in an image

How do we decide on a codebook

- Typically done with **clustering**

Vector Quantization: Example



Linear Transform Coding

Want to encode values over a region of time or space

- Typically used for images or audio

Select a set of linear basis functions ϕ_i that span the space

- sin, cos, spherical harmonics, wavelets, …
- Defined at discrete points

Linear Transform Coding

Coefficients: $\Theta_i = \sum_j x_j \phi_i(j) = \sum_j x_j a_{ij}$

$\Theta_i =$ i^{th} resulting coefficient

$x_j =$ j^{th} input value

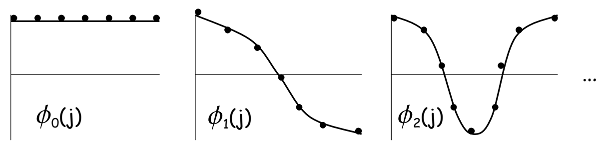
$a_{ij} =$ ij^{th} transform coefficient $= \phi_i(j)$

In matrix notation:

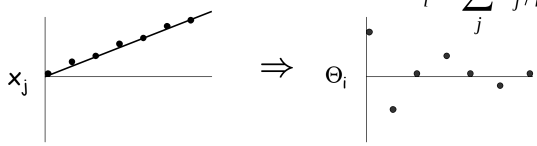
$$\Theta = Ax$$
$$x = A^{-1}\Theta$$

Where A is an n x n matrix, and each row defines a basis function

Example: Cosine Transform

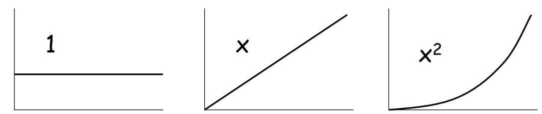


$$\Theta_i = \sum_j x_j \phi_i(j)$$

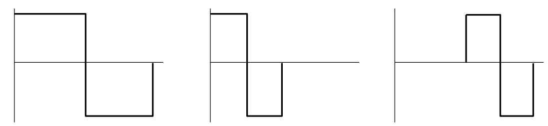


Other Transforms

Polynomial:



Wavelet (Haar):



How to Pick a Transform

Goals:

- Decorrelate
- Low coefficients for many terms
- Basis functions that can be ignored by perception

Why is using a Cosine of Fourier transform across a whole image bad?

How might we fix this?

Usefulness of Transform

Typically transforms A are **orthonormal**: $A^{-1} = A^T$

Properties of orthonormal transforms:

$\sum x^2 = \sum \Theta^2$ (energy conservation)

Would like to compact energy into as few coefficients as possible

$$G_{TC} = \frac{\frac{1}{n}\sum \sigma_i^2}{\left(\prod \sigma_i^2\right)^{1/n}}$$

(the **transform coding gain**)
arithmetic mean/geometric mean

$\sigma_i = (\Theta_i - \Theta_{av})$

The higher the gain, the better the compression

Case Study: JPEG

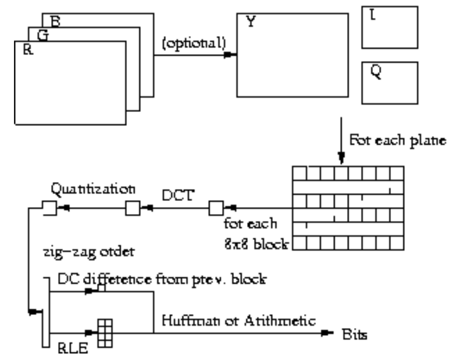
A nice example since it uses many techniques:

- Transform coding (Cosine transform)
- Scalar quantization
- Difference coding
- Run-length coding
- Huffman or arithmetic coding

JPEG (Joint Photographic Experts Group) was designed in **1991** for **lossy** and **lossless** compression of **color** or **grayscale images**. The lossless version is rarely used.

Can be adjusted for compression ratio (typically 10:1)

JPEG in a Nutshell

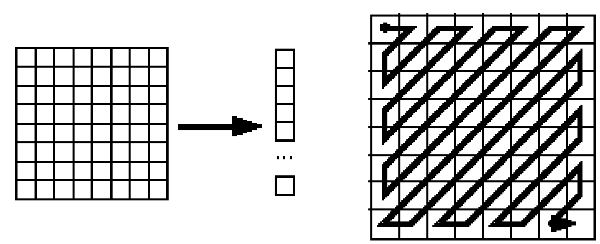


JPEG: Quantization Table

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Also divided through uniformaly by a quality factor which is under control.

JPEG: Block scanning order



Uses run-length coding for sequences of zeros

JPEG: example

.125 bits/pixel (factor of 200)

Case Study: MPEG

Pretty much JPEG with **interframe coding**

Three types of frames

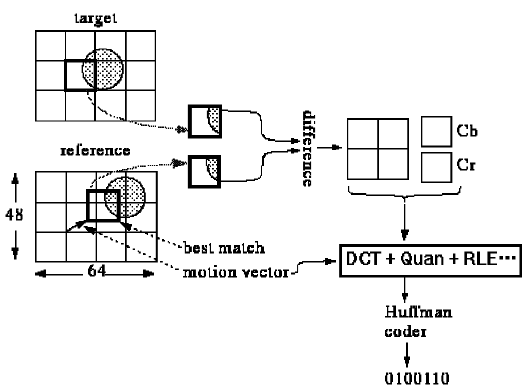
- **I** = intra frame (aprox. JPEG) anchors
- **P** = predictive coded frames
- B = bidirectionally predictive coded frames

Example:

Type:	I	B	B	P	B	B	P	B	B	P	B	B	I
Order:	1	3	4	2	6	7	5	9	10	8	12	13	11

I frames are used for random access.

MPEG matching between frames



MPEG: Compression Ratio

356 x 240 image

Type	Size	Compression
I	18KB	7/1
P	6KB	20/1
B	2.5KB	50/1
Average	4.8KB	27/1

30 frames/sec x 4.8KB/frame x 8 bits/byte
= 1.2 Mbits/sec + .25 Mbits/sec (stereo audio)

HDTV has 15x more pixels
= 18 Mbits/sec

MPEG in the "real world"

- DVDs
  - Adds "encryption" and error correcting codes
- Direct broadcast satellite
- HDTV standard
  - Adds error correcting code on top
- Storage Tech "Media Vault"
  - Stores 25,000 movies

Encoding is much more expensive than encoding.

Still requires special purpose hardware for high resolution and good compression.

Wavelet Compression

- A set of localized basis functions
- Avoids the need to block

"**mother function**" $\phi(x)$

$\phi_{sl}(x) = \phi(2^s x - l)$

s = scale l = location

Requirements

$$\int_{-\infty}^{\infty} \psi(x)dx = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} |\psi(x)|^2 dx < -\infty$$

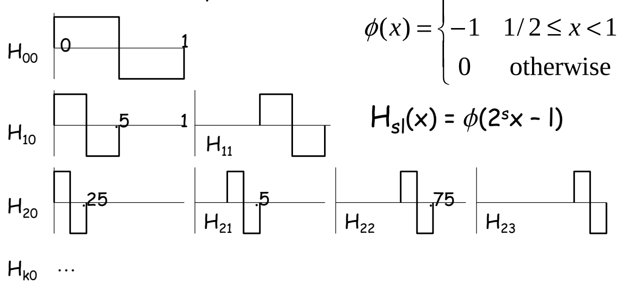
Many **mother** functions have been suggested.

Haar Wavelets

Most described, least used.

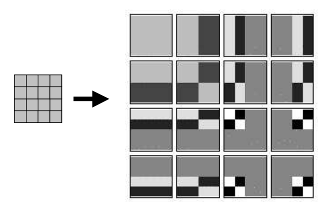
$$\phi(x) = \begin{cases} 1 & 0 \le x < 1/2 \\ -1 & 1/2 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$H_{sl}(x) = \phi(2^s x - l)$

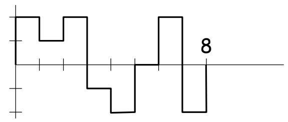


+ DC component = 2^{k+1} components

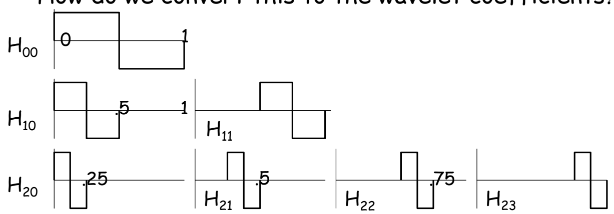
Haar Wavelet in 2d



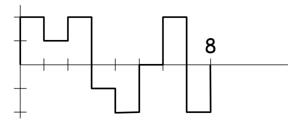
Discrete Haar Wavelet Transform



How do we convert this to the wavelet coefficients?



Discrete Haar Wavelet Transform



How do we convert this to the wavelet coefficients?

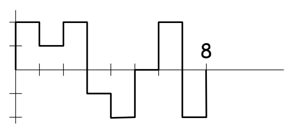
```
for (j = n/2; j >= 1; j = j/2) {
  for (i = 1; i < j; i++) {
    b[i] = (a[2i-1] + a[2i])/2;
    b[j+i] = (a[2i-1] – a[2i])/2; }
  a[1..2*j] = b[1..2*j]; }
```

Averages

Differences

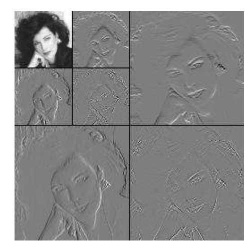
Linear time!

Haar Wavelet Transform: example



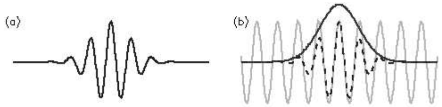
a	=	2	1	2	-1	-2	0	2	-2
	=	1.5	.5	-1	0	.5	1.5	-1	2
	=	1	-.5	.5	-.5				
	=	.25	.75						
a	=	.25	.75	.5	.5	.5	1.5	-1	2

Wavelet decomposition



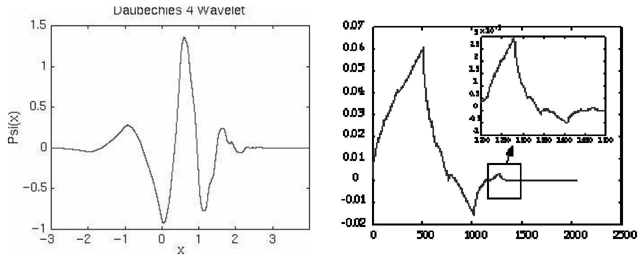
Morlet Wavelet

$\phi(x)$ = Gaussian · Cosine = $e^{-(x^2/2)}\cos(5x)$



Corresponds to wavepackets in physics.

Daubechies Wavelet



JPEG2000

Overall Goals:

- High compression efficiency with good quality at compression ratios of .25bpp
- Handle large images (up to 2^{32} x 2^{32})
- Progressive image transmission
  - Quality, resolution or region of interest
- Fast access to various points in compressed stream
- Pan and Zoom while only decompressing parts
- Error resilience

JPEG2000: Outline

Main similarities with JPEG

- Separates into Y, I, Q color planes, and can downsample the I and Q planes
- Transform coding

Main differences with JPEG

- Wavelet transform
  - Daubechies 9-tap/7-tap (irreversible)
  - Daubechies 5-tap/3-tap (reversible)
- Many levels of hierarchy (resolution and spatial)
- Only arithmetic coding

JPEG2000: 5-tap/3-tap

```
h[i] = a[2i-1] - (a[2i] + a[2i-2])/2;
l[i] = a[2i] + (h[i-1] + h[i] + 2)/2;
```

`h[i]`: is the "high pass" filter, ie, the **differences**
it depends on 3 values from a (3-tap)

`l[i]`: is the "low pass" filter, ie, the **averages**
it depends on 5 values from a (5-tap)

Need to deal with boundary effects.

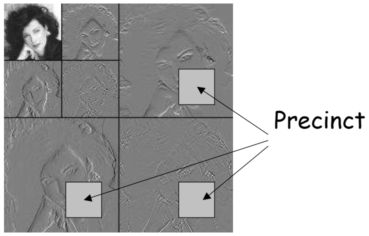
This is reversible: assignment

JPEG 2000: Outline

A spatial and resolution hierarchy

- **Tiles:** Makes it easy to decode sections of an image. For our purposes we can imagine the whole image as one tile.
- **Resolution Levels:** These are based on the wavelet transform. High-detail vs. Low detail.
- **Precinct Partitions:** Used within each resolution level to represent a region of space.
- **Code Blocks:** blocks within a precinct
- **Bit Planes:** ordering of significance of the bits

JPEG2000: Precincts



JPEG vs. JPEG2000

JPEG: .125bpp

JPEG2000: .125bpp

Compression Outline

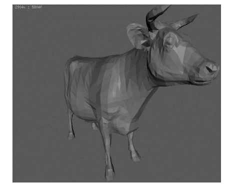
Introduction: Lossy vs. Lossless, Benchmarks, …
Information Theory: Entropy, etc.
Probability Coding: Huffman + Arithmetic Coding
Applications of Probability Coding: PPM + others
Lempel-Ziv Algorithms: LZ77, gzip, compress, …
Other Lossless Algorithms: Burrows-Wheeler
Lossy algorithms for images: JPEG, MPEG, ...
Compressing graphs and meshes: BBK

Compressing Structured Data

So far we have concentrated on Text and Images, compressing sound is also well understood.

What about various forms of "structured" data?

- Web indexes
- Triangulated meshes used in graphics
- Maps (mapquest on a palm)
- XML
- Databases



Compressing Graphs

<u>**Goal:**</u> To represent large graphs compactly while supporting queries efficiently

- e.g., adjacency and neighbor queries

Applications:

- Large web graphs
- Large meshes
- Phone call graphs

Results

O(n)-bit compression on "separable graphs" with n vertices, with O(1) access time

Experimental results:

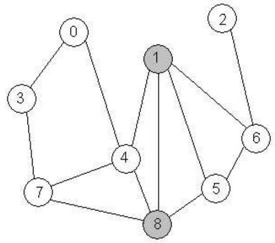
- 6-12 bits/edge in total
- access time faster than linked-list representation

Vertex Separators

A *vertex separator* for G=(V,E) is a set of vertices $U \subset V$, that partitions V/U into two components V_1 and V_2

A class of graphs S satisfies a *f(n)-vertex separator theorem* if
$\exists\ \alpha < 1, \beta > 0$
$\forall (V,E) \in S, \exists$ separator U,
$|U| < \beta\ f(|V|)$,
$|V_i| < \alpha|V|$, i = 1,2

A *separable class of graphs* is one that satisfies an n^c-separator theorem, c<1.

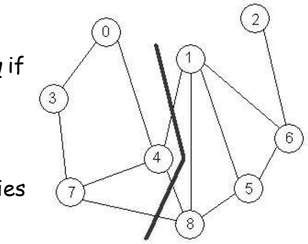


Edge Separators

An *edge separator* for (V,E) is a set of edges $E' \subset E$ whose removal partitions V into two components V_1 and V_2

A class of graphs S satisfies a *f(n)-edge separator theorem* if
$\exists\ \alpha < 1, \beta > 0$
$\forall (V,E) \in S, \exists$ separator E',
$|E'| < \beta\ f(|V|)$,
$|V_i| < \alpha|V|$, i = 1,2

Any class of graphs that satisfies an edge separator theorem satisfies the corresponding vertex separator theorem as well.



Separable Classes of Graphs

Planar graphs: $O(n^{1/2})$ separators

Well-shaped meshes in R^d: $O(n^{1-1/d})$ [Miller et al.]

Nearest-neighbor graphs

In practice, good separators from circuit graphs, street graphs, web connectivity graphs, router connectivity graphs

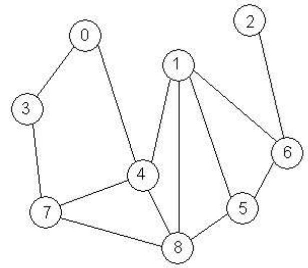
Note: All separable classes of graphs have bounded density (m is O(n))

Main Ideas

For good edge separators
- Number vertices using separators
- Use difference coding on adjacency lists
- Using efficient data structure for indexing

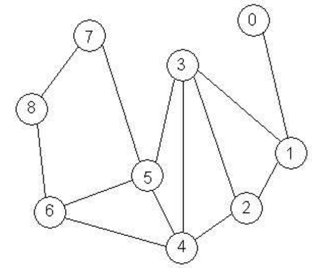
For good vertex separators
- Each vertex assigned multiple labels
- Separate "root-find" data structure to map labels to a representative
- For adjacency queries, may need to direct graph

Compressed Adjacency Tables



#	D	Neighbors	Differences
0	2	3 4	3 1
1	4	4 5 6 8	3 1 1 2
2	1	6	4
3	2	0 7	-3 7
4	4	0 1 7 8	-4 1 6 1
5	3	1 6 8	-4 5 2
6	3	1 2 5	-5 1 3
7	3	3 4 8	-4 1 4
8	4	1 4 5 7	-7 3 1 2

Compressed Adjacency Tables



#	D	Neighbors	Differences
0	1	1	1
1	3	0 2 3	-1 2 1
2	3	1 3 4	-1 2 1
3	4	1 2 4 5	-1 1 2 1
4	4	2 3 5 6	-2 1 2 1
5	4	3 4 6 7	-2 1 2 1
6	3	4 5 8	-2 1 3
7	2	5 8	-2 3
8	2	6 7	-2 1

Log-sized Codes

Log-sized code: Any prefix code that takes O(log(d)) bits to represent an integer d.
Gamma code, delta code, skewed Bernoulli code

Example: Gamma code
Prefix: unary code for $\lfloor \log d \rfloor$
Suffix: binary code for $d-2^{\lfloor \log d \rfloor}$
(binary code for d, except leading 1 is implied)

Decimal	Gamma	
1	1	
2	01	0
3	01	1
4	001	00
5	001	01
6	001	10
7	001	11
8	0001	000

Difference Coding

For each vertex, encode:
- Degree
- Sign of first entry
- Differences in adjacency list

Concatenate vertex encodings to encode the graph

#	D	Differences
0	2	3 1

010 0 011 1
degree sign 3 1

#	D	Differences
4	4	-4 1 6 1

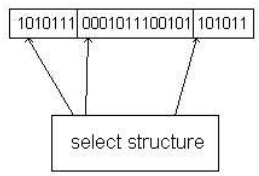
00100 1 00100 1 00110 1
degree sign 4 1 6 1

Indexing

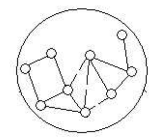
Problem: We need to find the start of a vertex quickly.

Formally: Given n numbers in range 1..O(n), prepare a data structure that returns the k^{th} smallest number.

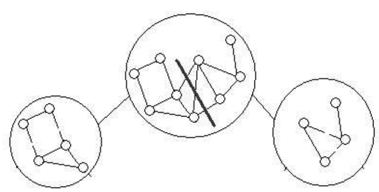
Can be supported with O(n) bits and O(1) access time.



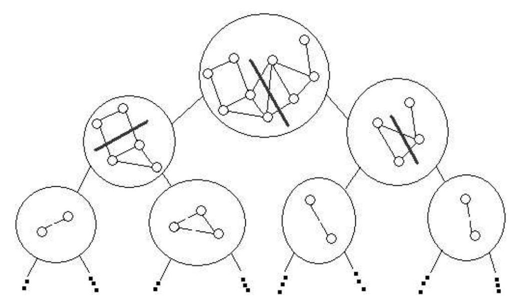
Renumbering with Edge Separators



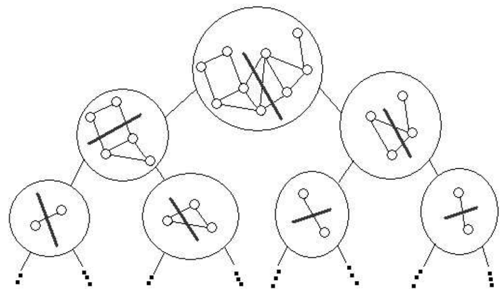
Renumbering with Edge Separators



Renumbering with Edge Separators



Renumbering with Edge Separators



Theorem (edge separators)

Any class of graphs that allows $O(n^c)$ edge separators can be compressed to $O(n)$ bits with $O(1)$ access time using:

- Difference coded adjacency lists
- $O(n)$-bit indexing structure

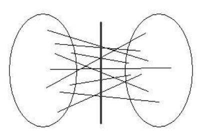
Proof Outline

Bound cost of adjacency lists: cost of edge (a,b) is at most $O(\log(|a-b|))$

If an edge is a separator in a graph of size d, then its cost is at most log(d)

n^c edges in separator: cost $n^c \log(n)$

$T(n)=n^c \log(n) + 2T(n/2)$
$= O(n)$



Experimental Results: Test Graphs

Graph	Vtxs	Edges	Max Degree	Source
auto	448695	3314611	37	3D mesh [28]
feocean	143437	409593	6	3D mesh [28]
m14b	214765	1679018	40	3D mesh [28]
ibm17	185495	2235716	150	circuit [2]
ibm18	210613	2221860	173	circuit [2]
CA	1971281	2766607	12	street map [27]
PA	1090920	1541898	9	street map [27]
googleI	916428	5105039	6326	web links [9]
googleO	916428	5105039	456	web links [9]
lucent	112969	181639	423	routers [24]
scan	228298	320168	1937	routers [24]

Performance: Adjacency Table

	dfs		metis-cf		bu-bpq		bu-cf	
	T_d	Space	T/T_d	Space	T/T_d	Space	T/T_d	Space
auto	0.79	9.88	153.11	5.17	7.54	5.90	14.59	5.52
feocean	0.06	13.88	388.83	7.66	17.16	8.45	34.83	7.79
m14b	0.31	10.65	181.41	4.81	8.16	5.45	15.32	5.13
ibm17	0.44	13.01	136.43	6.18	11.0	6.79	20.25	6.64
ibm18	0.48	11.88	129.22	5.72	9.5	6.24	17.29	6.13
CA	0.76	8.41	382.67	4.38	14.61	4.90	35.21	4.29
PA	0.43	8.47	364.06	4.45	13.95	4.98	33.02	4.37
googleI	1.4	7.44	186.91	4.08	12.71	4.18	40.96	4.14
googleO	1.4	11.03	186.91	6.78	12.71	6.21	40.96	6.05
lucent	0.04	7.56	390.75	5.52	19.5	5.54	45.75	5.44
scan	0.12	8.00	280.25	5.94	23.33	5.76	81.75	5.66
Avg		10.02	**252.78**	5.52	**13.65**	5.86	**34.54**	5.56

Time is to create the structure, normalized to time for DFS

Performance: Overall

	Array		List		bu-cf/semi	
Graph	time	space	time	space	time	space
auto	0.24	34.2	0.61	66.2	0.51	7.17
feocean	0.04	37.6	0.08	69.6	0.09	11.75
m14b	0.11	34.1	0.29	66.1	0.24	6.70
ibm17	0.15	33.3	0.40	65.3	0.34	7.72
ibm18	0.14	33.5	0.38	65.5	0.32	7.33
CA	0.34	43.4	0.56	75.4	0.58	11.66
PA	0.19	43.3	0.31	75.3	0.32	11.68
googleI	0.24	37.7	0.49	69.7	0.45	7.86
googleO	0.24	37.7	0.50	69.7	0.51	9.90
lucent	0.02	42.0	0.04	74.0	0.05	11.87
scan	0.04	43.4	0.06	75.4	0.08	12.85

`time is for one DFS`

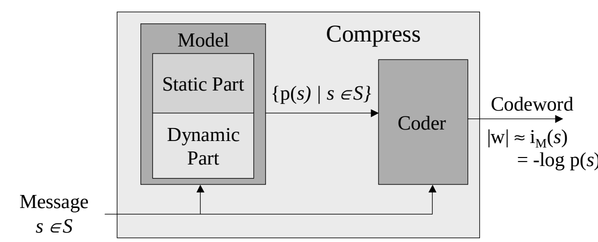
Conclusions

O(n)-bit representation of separable graphs with O(1)-time queries

Space efficient and fast in practice for a wide variety of graphs.

Compression Summary

Compression is all about **probabilities**



We want the model to skew the probabilities as much as possible (*i.e.*, decrease the **entropy**)

Compression Summary

How do we figure out the probabilities
- Transformations that skew them
  - Guess value and code difference
  - Move to front for temporal locality
  - Run-length
  - Linear transforms (Cosine, Wavelet)
  - Renumber (graph compression)
- Conditional probabilities
  - Neighboring context

In practice one almost always uses a combination of techniques