Algorithms in the Real World
Assignment #4 — Due Date: 19 Oct 03

Do any four of the following five problems.

Problem 1: (25pt) (Strang Chapter 8 problem 8.2.1)

$$
\begin{array}{llllllll}
\text{minimize:} & x_1 & +x_2 & -x_3 & & & & \\
\text{subject to:} & 2x_1 & -4x_2 & +x_3 & +x_4 & & = 4 \\
 & 3x_1 & +5x_2 & +x_3 & & +x_5 & = 2 \\
 & & & & & x & \geq 0
\end{array}
$$

Which of x_1, x_2, x_3 should enter the basis, an which of x_4 and x_5 should leave? Compute the new pair of basic variables and find the cost at the new corner.

Problem 2: (25pt)
Consider the following problem

$$
\begin{array}{ll}
\text{maximize} & x_1 + x_2 \\
\text{subject to} & 2x_1 + x_2 \leq 4 \\
 & x \geq 0
\end{array}
$$

(a) Starting with the initial solution $(x_1, x_2) = (.1, .1)$ solve this problem using the affine scaling interior-point method. You should choose a step size that moves you 96% of the way from your current location to the boundary of the feasible region.

(b) Illustrate the progress of the algorithm on a graph in $x_1 - x_2$ space. Note that the solution should only be taking a few steps to get very close to the final solution.

Problem 3: (25pt)
Let's say we augment linear programs to allow constraints to include absolute values (*e.g.* $|x_1| + 3|x_2| \leq b$). Can we solve all such problems in polynomial time? Show why or why not. (Assume $P \neq NP$.)

Problem 4: (25pt)
Let $S = \{x \in R^n : Ax \leq b\}$ and $T = \{x \in R^n : Bx \leq d\}$. Assuming that S and T are nonempty, describe a polynomial time algorithm (in n) for checking whether $S \subset T$.

Problem 5: (25pt)
Solve the following problem using the implicit enumeration branch-and-bound technique (the 0-1 programming technique) describe in class. You should show the tree and mark on any leaf node why it was pruned.

$$
\begin{array}{ll}
\text{minimize} & 3x_1 + 2x_2 + 5x_3 + x_4 \\
\text{subject to} & -2x_1 + x_2 - x_3 - 2x_4 \leq -2 \\
 & -x_1 - 5x_2 - 2x_3 + 3x_4 \leq -3 \\
 & x_1, x_2, x_3, x_4 \text{ binary}
\end{array}
$$